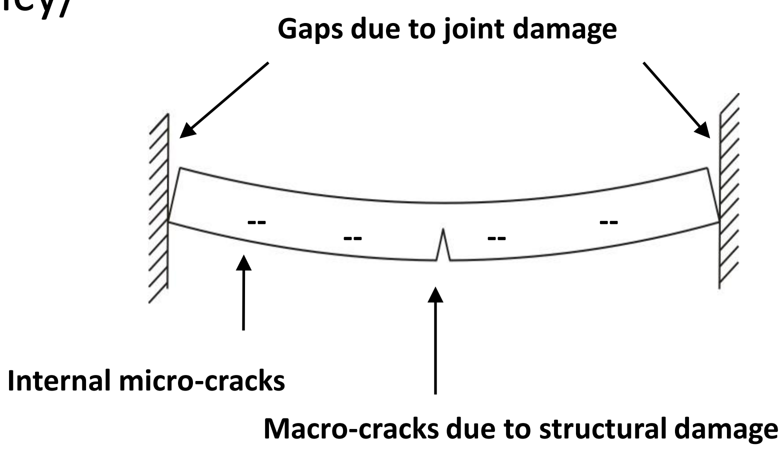
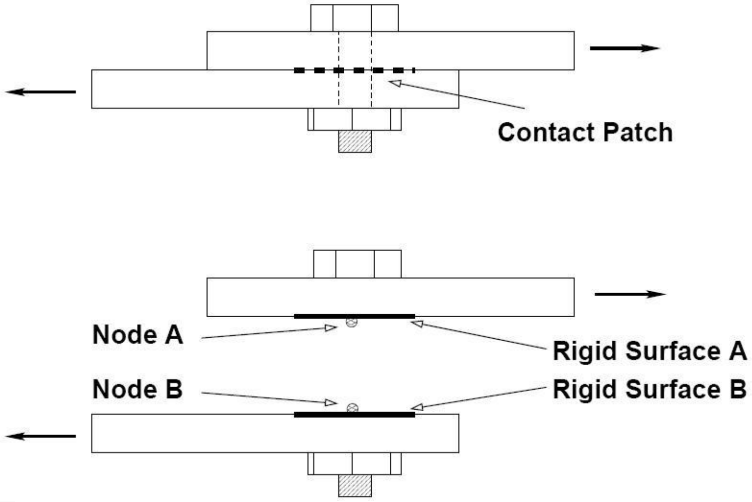
Types of structural damage

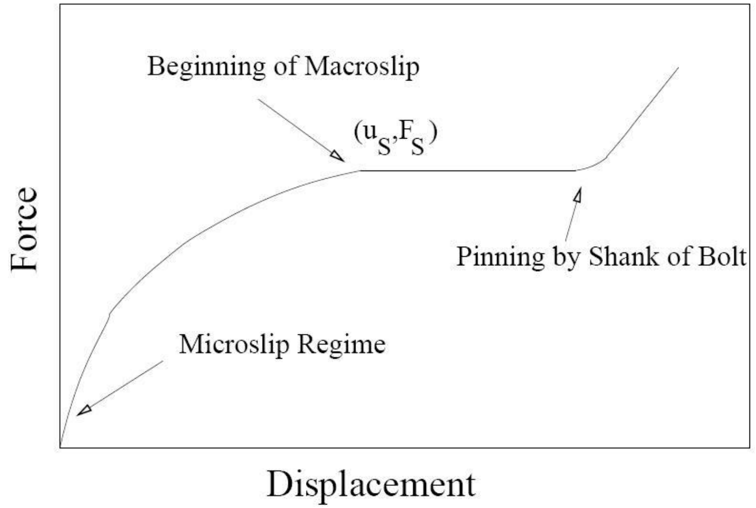
- Internal micro-cracks change elastic properties (global vs. local)
- Internal macro-cracks change local elastic properties, and hence the natural vibration frequencies and mode shapes
- Joint damage changes elastic properties at boundaries, frequency/ mode shapes, & adds damping
- Fundamental frequencies are lowered while adding higher harmonics



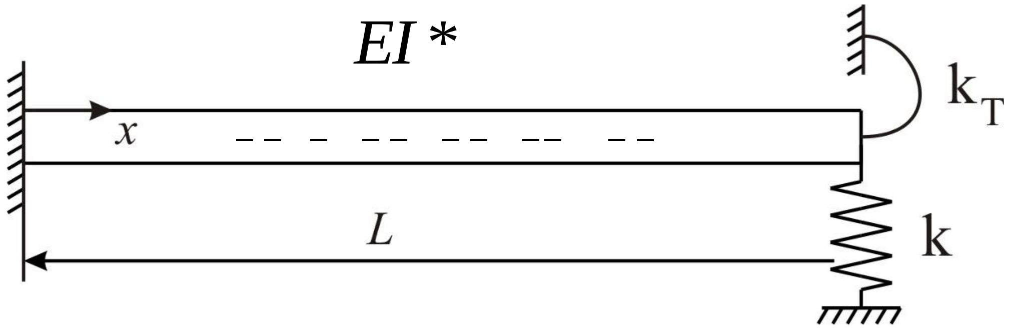
Dynamic Effects of Damaged Joints

- A shift (usually a decrease) in the natural frequencies of the structure.
- An increase in structural damping due to the physics of micro-slip, friction.
- The introduction of nonlinear effects due to vibro-impact.
- The reflection and transmission of elastic waves across the joint .





Example: Beam with both Internal (micro-) damage and Boundary Damage



The internal damage is represented by a lower average rigidity EI*<EI.

The damaged boundary is represented by the translational and torsional springs.

Define nondimensional boundary damage ratios of the damaged internal stiffness to the damaged boundary stiffness, e.g.

$$\alpha = \frac{EI^*}{kL^3} \qquad \alpha_T = \frac{EI^*}{k_T L}$$

Methods to Obtain Frequencies of Beam with Damaged Boundary

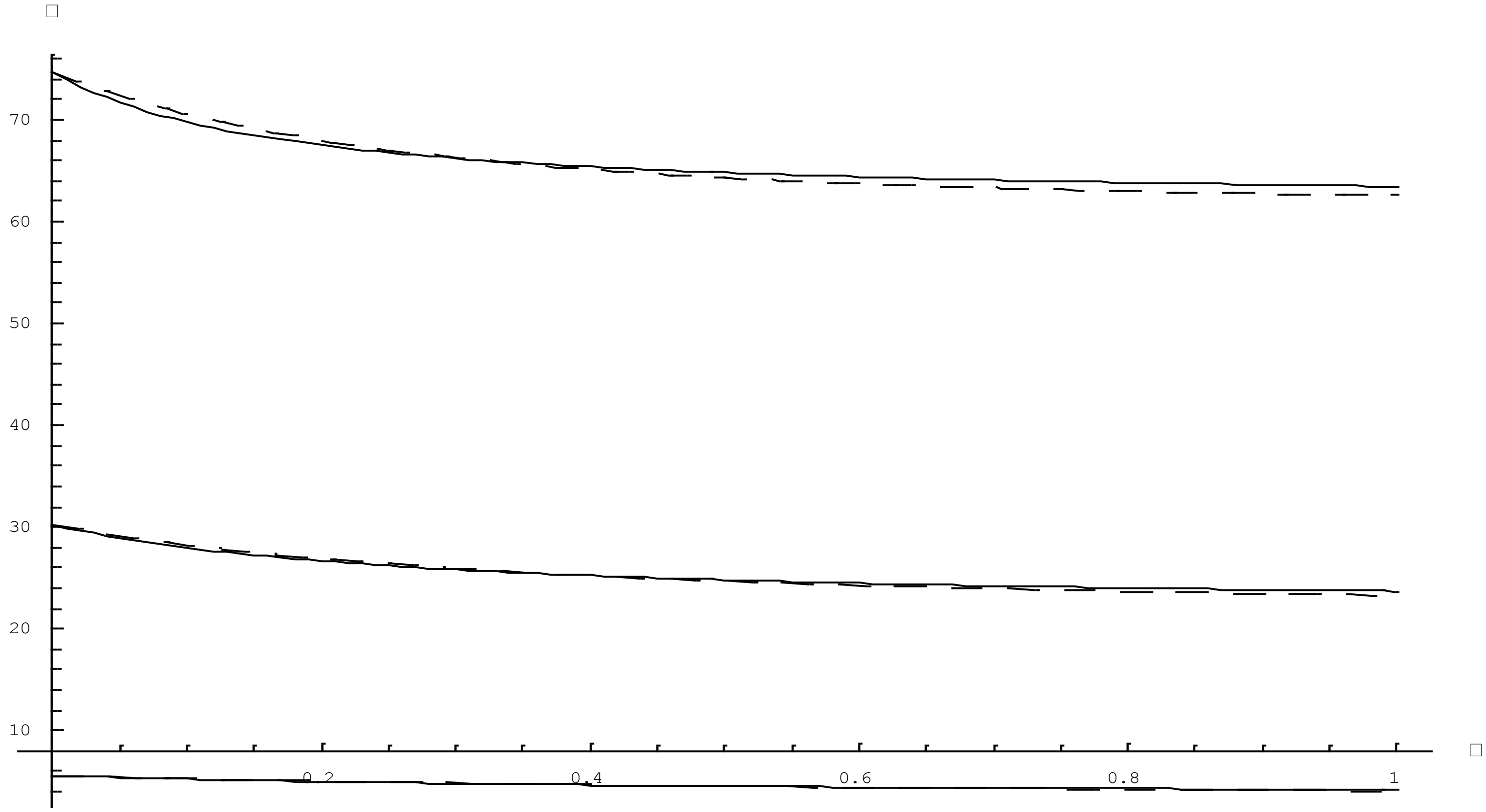
1. Interpolating between the natural frequencies of the clamped-fixed beam and clamped-free beam.

2. Minimizing the weighted integral squared error between the exact and approximate frequencies

3. Expansion in shifted Chebyshev polynomials to obtain an algebraic eigenvalue problem.

4. Perturbation method with Pade approximation.

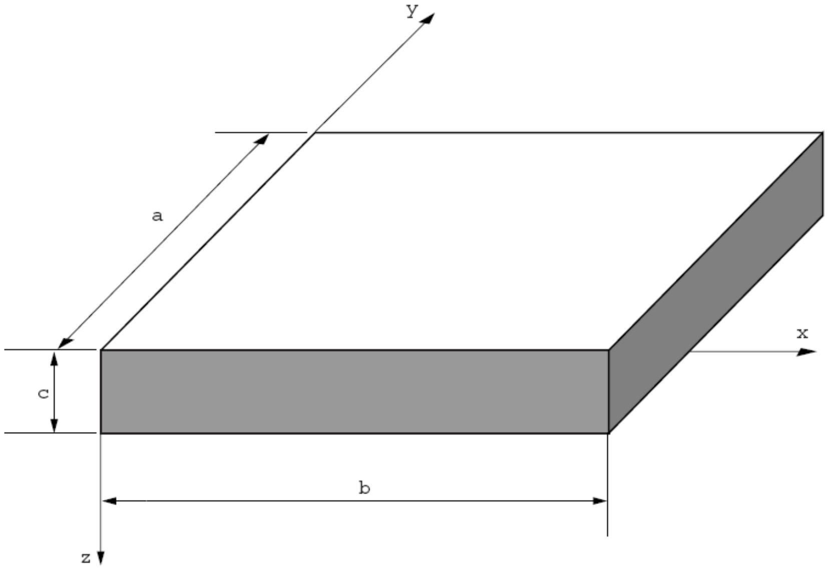
Natural Frequencies versus the translational boundary damage parameter



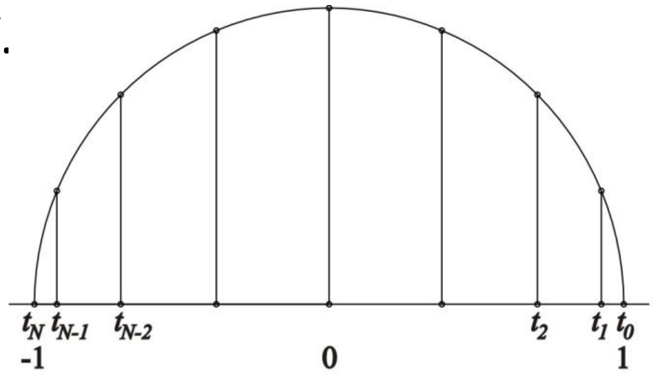
First three natural transverse vibrations frequencies (rad/s) versus the boundary damage parameter α (solid : exact, dashed : interpolated)

Three-Dimensional Vibrations For Thick Plates

- Three methods can be used in combination with the linear, three dimensional elasticity theory:



1. A polynomial-based Ritz energy approach.
2. The differential quadrature method.
3. The *Chebyshev spectral collocation method*. (currently being implemented)

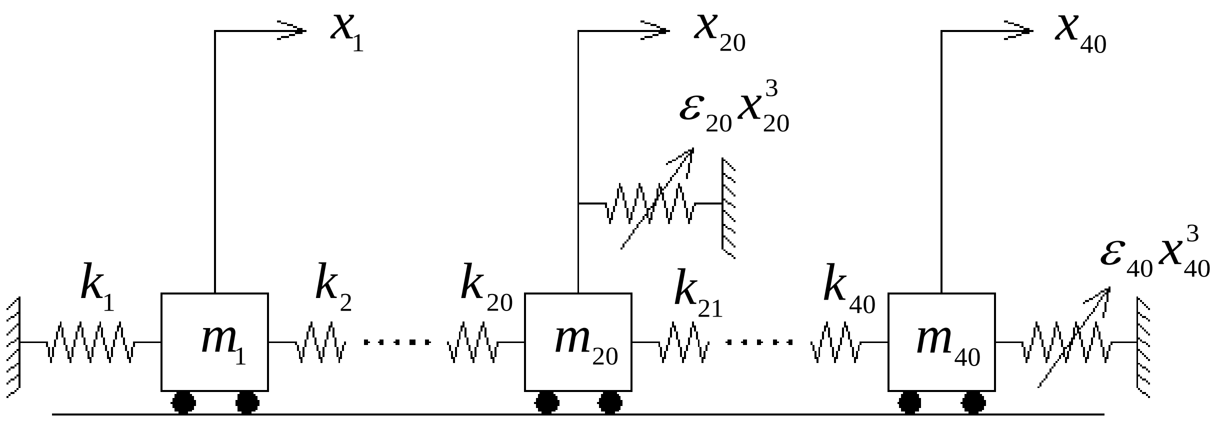


Relevance to structural health monitoring :

1. Small instruments using for measurement/ data collection
2. Allows a focus on a limited portion of the frequency spectrum
3. Model updating due to damage
4. Controlling a damaged structure (vibration localization)

Example : 40-DOF Mass-Spring System with Two Cubic Nonlinearities

$40-dof$ mass-spring system, $f(x_{40}) = \varepsilon_{40} x_{40}^3$ and $f(x_{20}) = \varepsilon_{20} x_{20}^3$



The parameters are

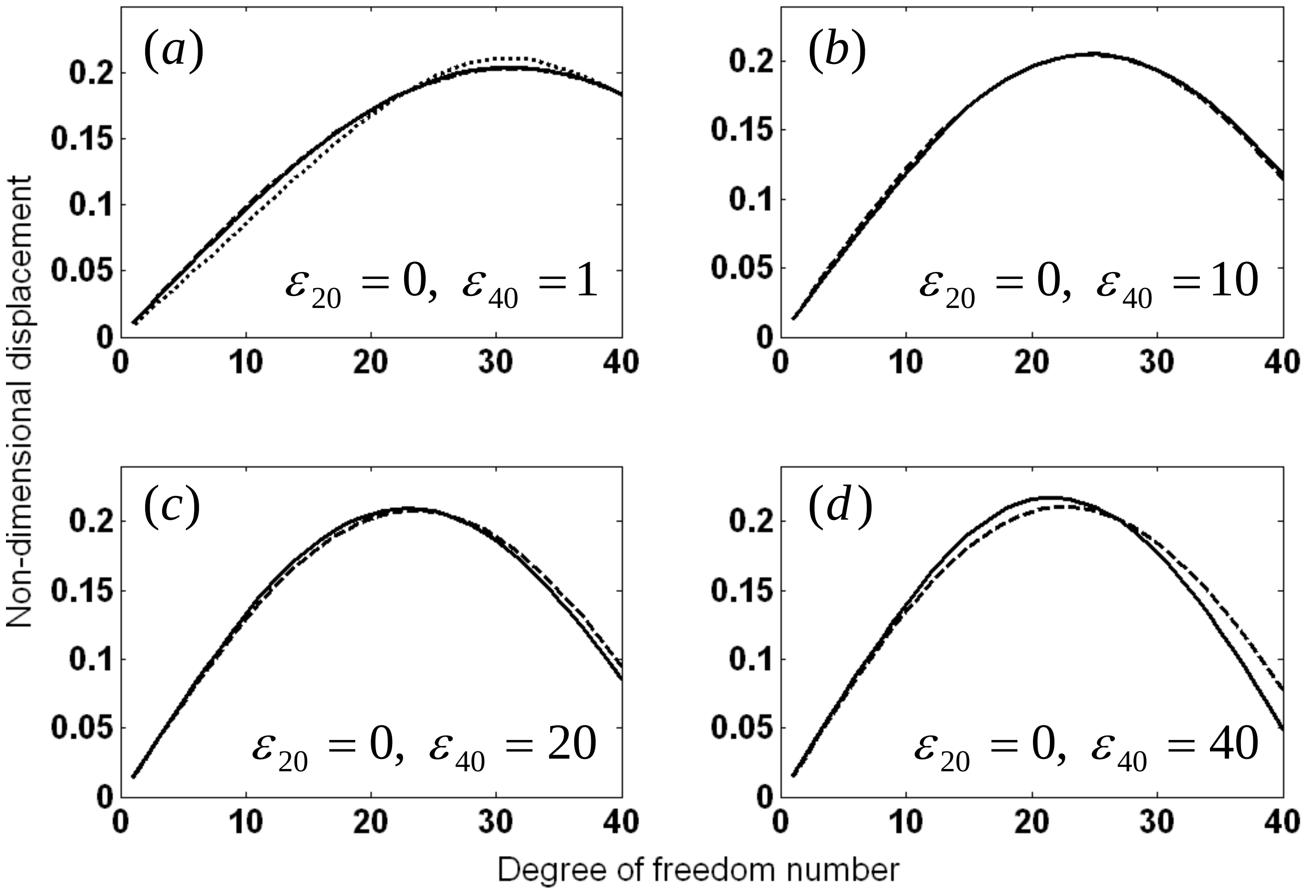
$$\varepsilon_{20} = \varepsilon_{40} = 3,\ n = 40$$

$$m_1 = m_2 = \ldots = m_{40} = 1$$

$$k_1 = k_2 = \ldots = k_{40} = 1$$

Iterated LELSM and Principal Orthogonal Modes of 40-DOF System with One Cubic Springs

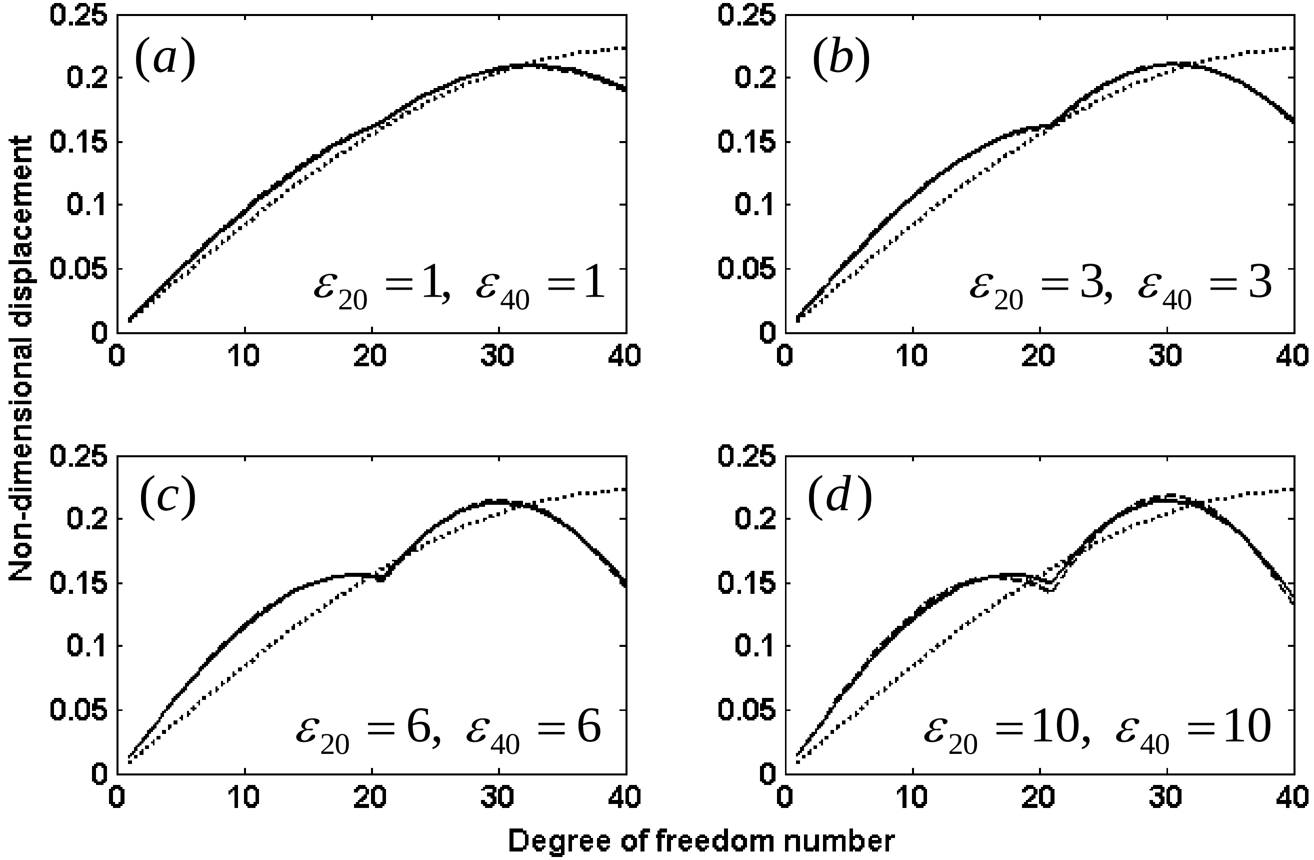
One cubic spring



First POM (solid), first iterated LELSM mode (6 iterations-dashed)
1st NNM mode (dots)

Iterated LELSM and Principal Orthogonal Modes of 40-DOF System with Two Cubic Springs

Two cubic springs

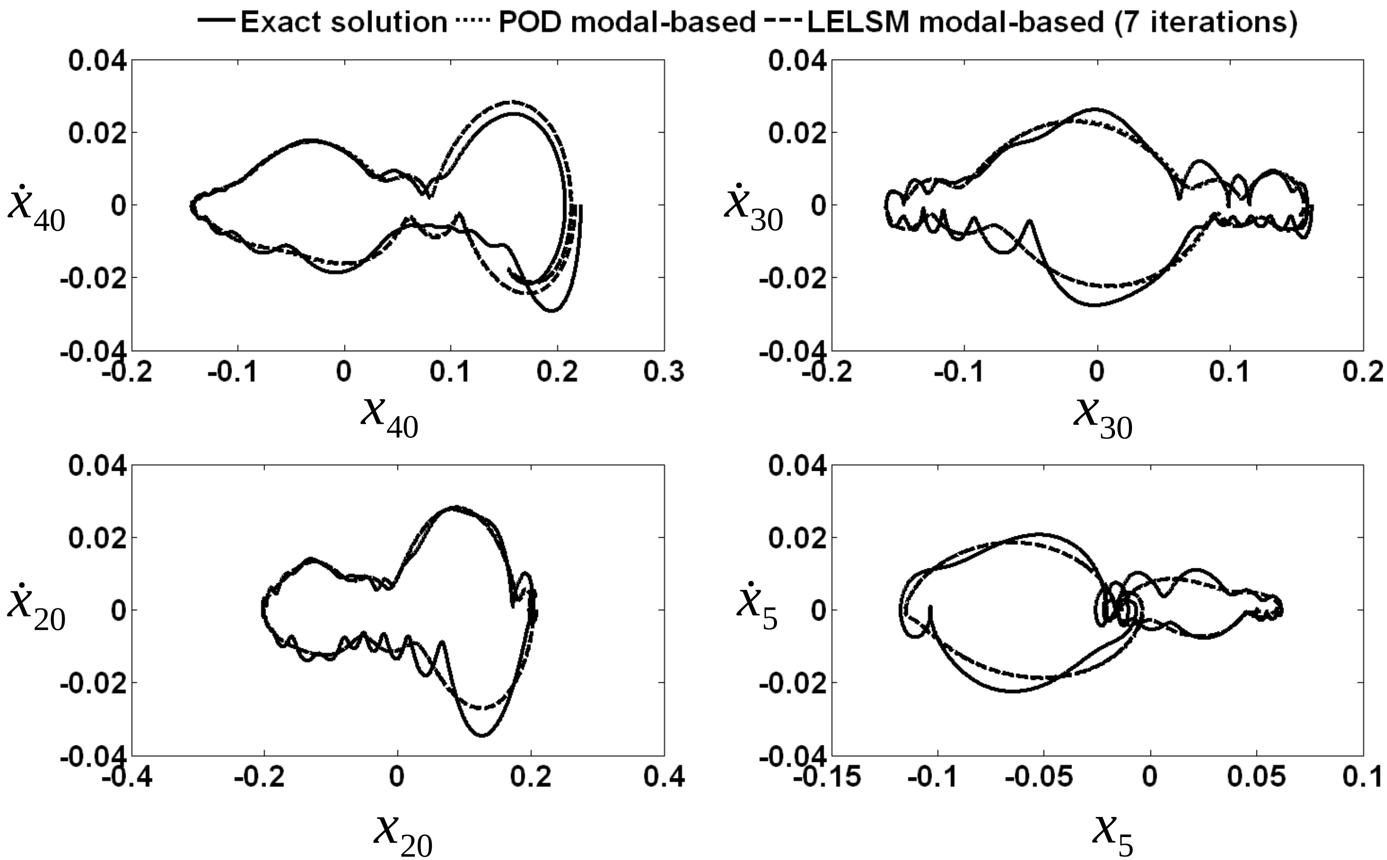


First POM (solid), first iterated LELSM mode (6 iterations-dashed)

Initial condition (1st linear mode-dots)

LELSM Modal-Based Order Reduction of 40-DOF System with Two Cubic Springs-Undamped

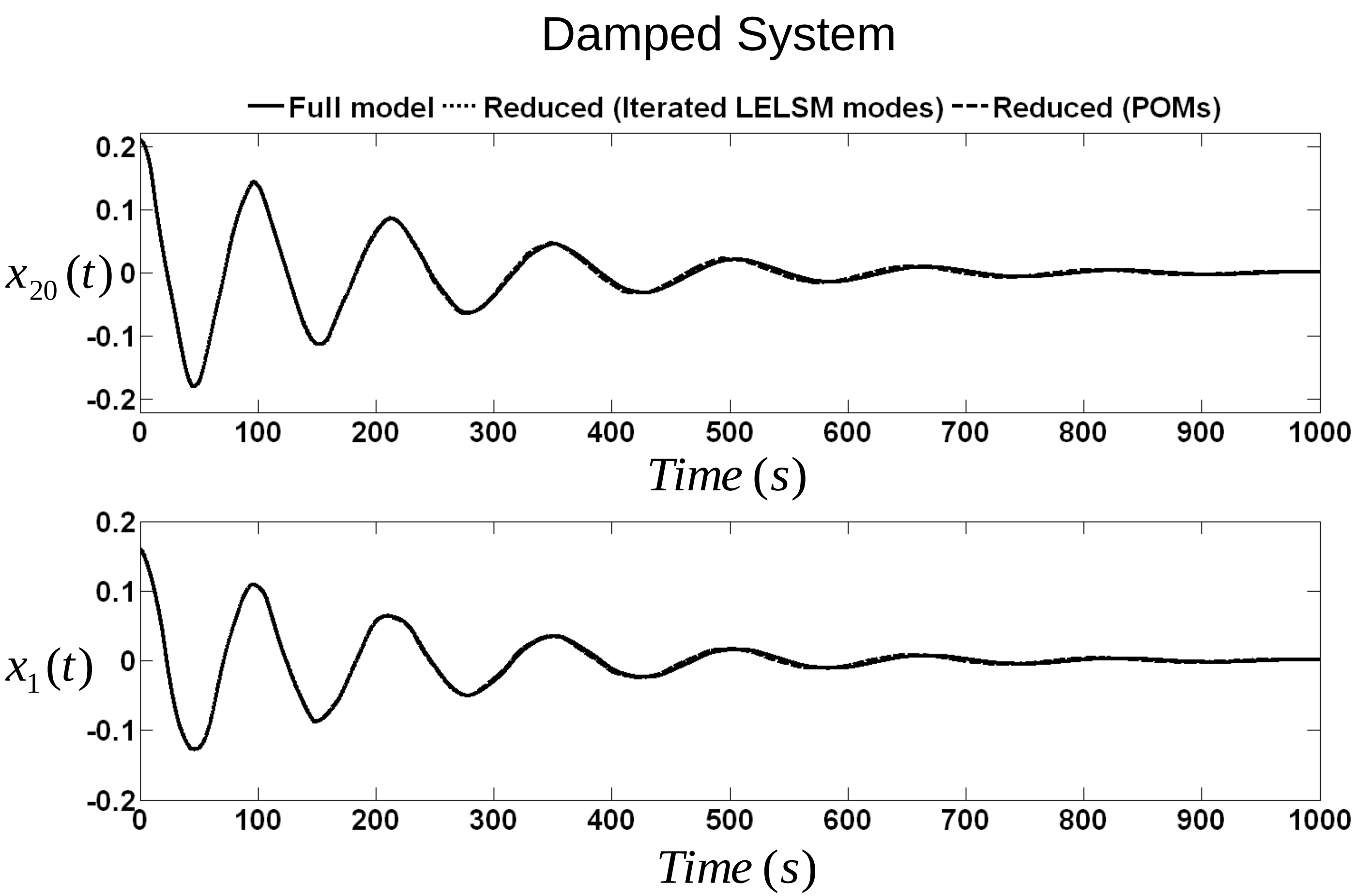
Undamped System



$$\varepsilon_{20} = 3,\ \varepsilon_{40} = 3.\ \mathrm{x}(0) = \phi_1^{Linear},\ \dot{\mathrm{x}}(0) = 0$$

Both LELSM and POD reduced models use 10 modes

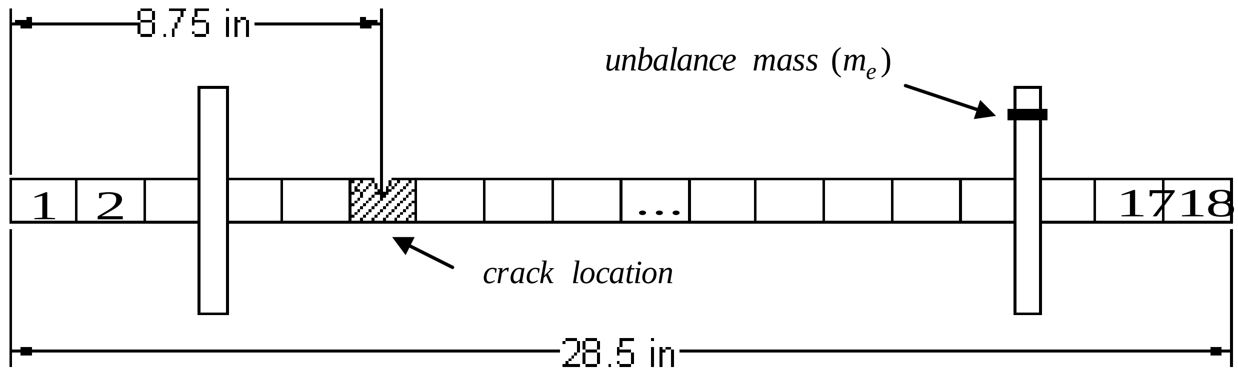
LELSM Modal-Based Order Reduction of 40-DOF System with Two Cubic Springs-Damped



$$\varepsilon_{20} = 3,\ \varepsilon_{40} = 3.\ \mathrm{x}(0) = \phi_1^{LELSM},\ \dot{\mathrm{x}}(0) = 0$$

Both LELSM and POD reduced models use 10 modes

Damage Detection in Rotor-Bearing-Disk system
Crack Models: Breathing and Open Crack Models



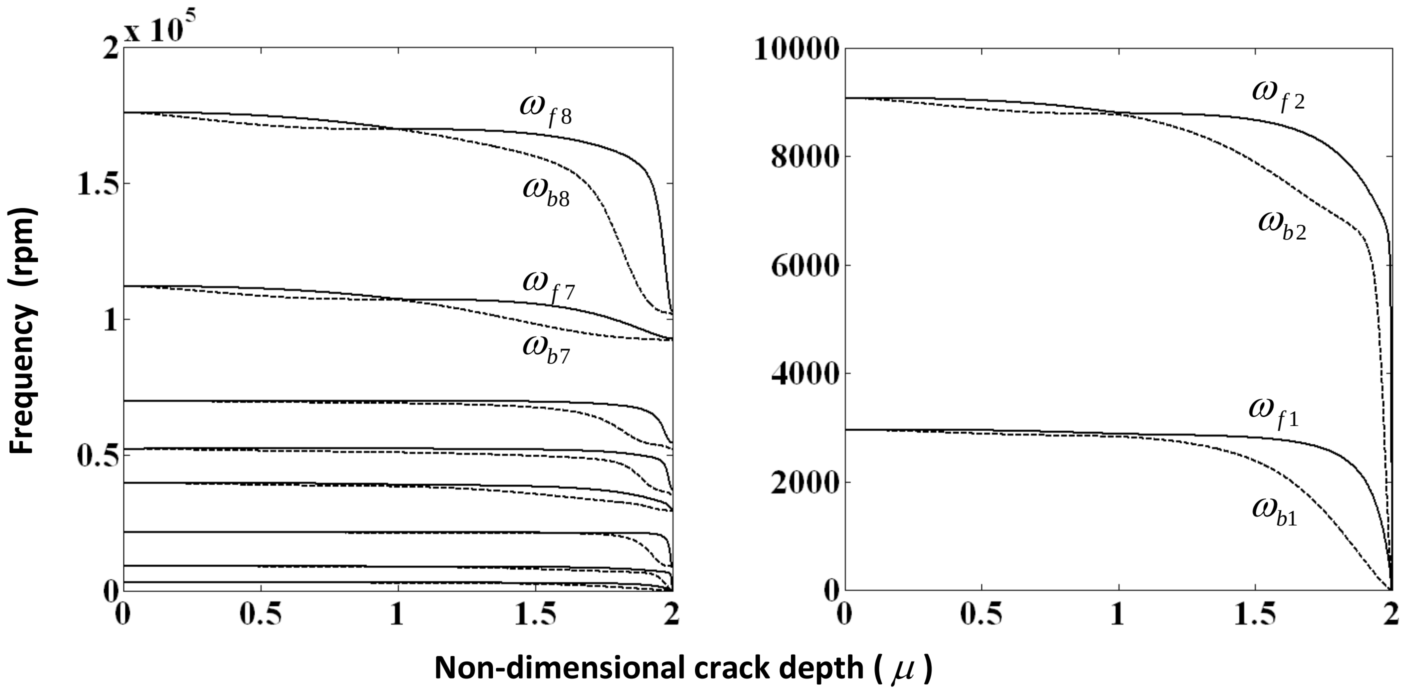
Finite element model of rotor-disk-bearing system

TABLE I PHYSICAL PARAMETERS OF THE MFS-RDS ROTOR-DYNAMIC SIMULATOR

Description	Value	Description	Value
Length of the rotor, L	$28.5\ in$	Disk outer radius, R_o	$3\ in$
Radius of the rotor, R	$0.625\ in$	Disk Inner radius, R_i	$0.625\ in$
Density of rotor, ρ	$7800\ kg/m^3$	Density of disk, ρ	$2700\ kg/m^3$
Modulus of elasticity, E	$2.1\times10^{11}\ N/m^2$	Mass of the disk, m_d	$0.571\ kg$
Bearing stiffness, (k_{xx}, k_{yy})	$7\times10^7\ N/m$	Mass unbalance, $m_e d$	$10^{-7}\ kg.m$
Bearing damping, (c_{xx}, c_{yy})	$5\times10^2\ Ns/m$	Mass unbalance angle, α	$0\ rad$

Damage Detection in Rotor-Bearing-Disk system
Crack Model: Open Crack Model

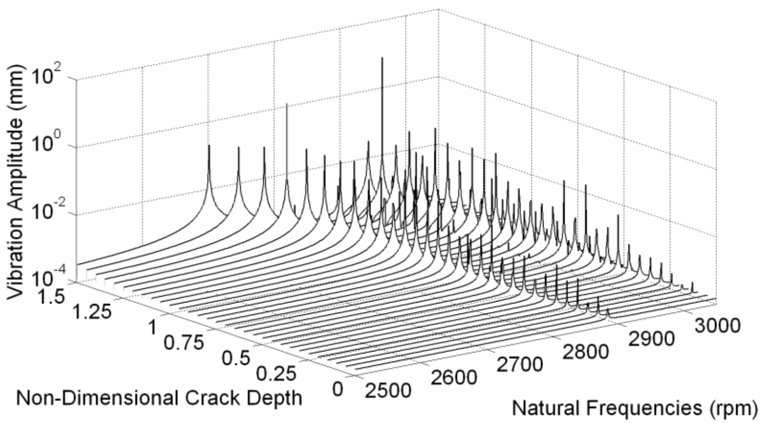
Open crack model



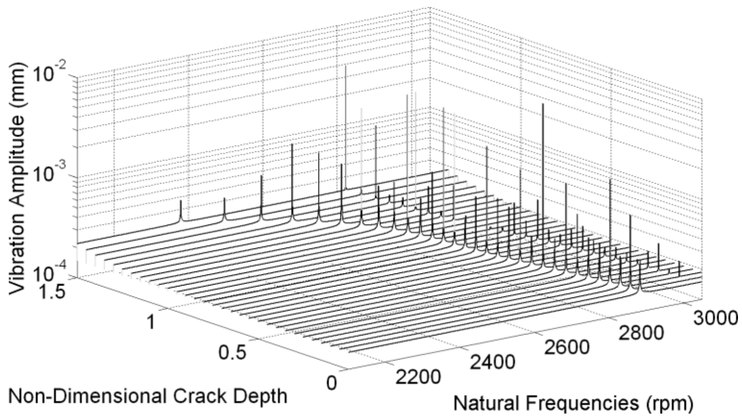
Veering phenomena in the first eight pairs of the natural frequencies of the forward whirl (solid) and the backward whirl (dashed)

Damage Detection in Rotor-Bearing-Disk system
Crack Models: Breathing and Open Crack Models

Waterfall plots

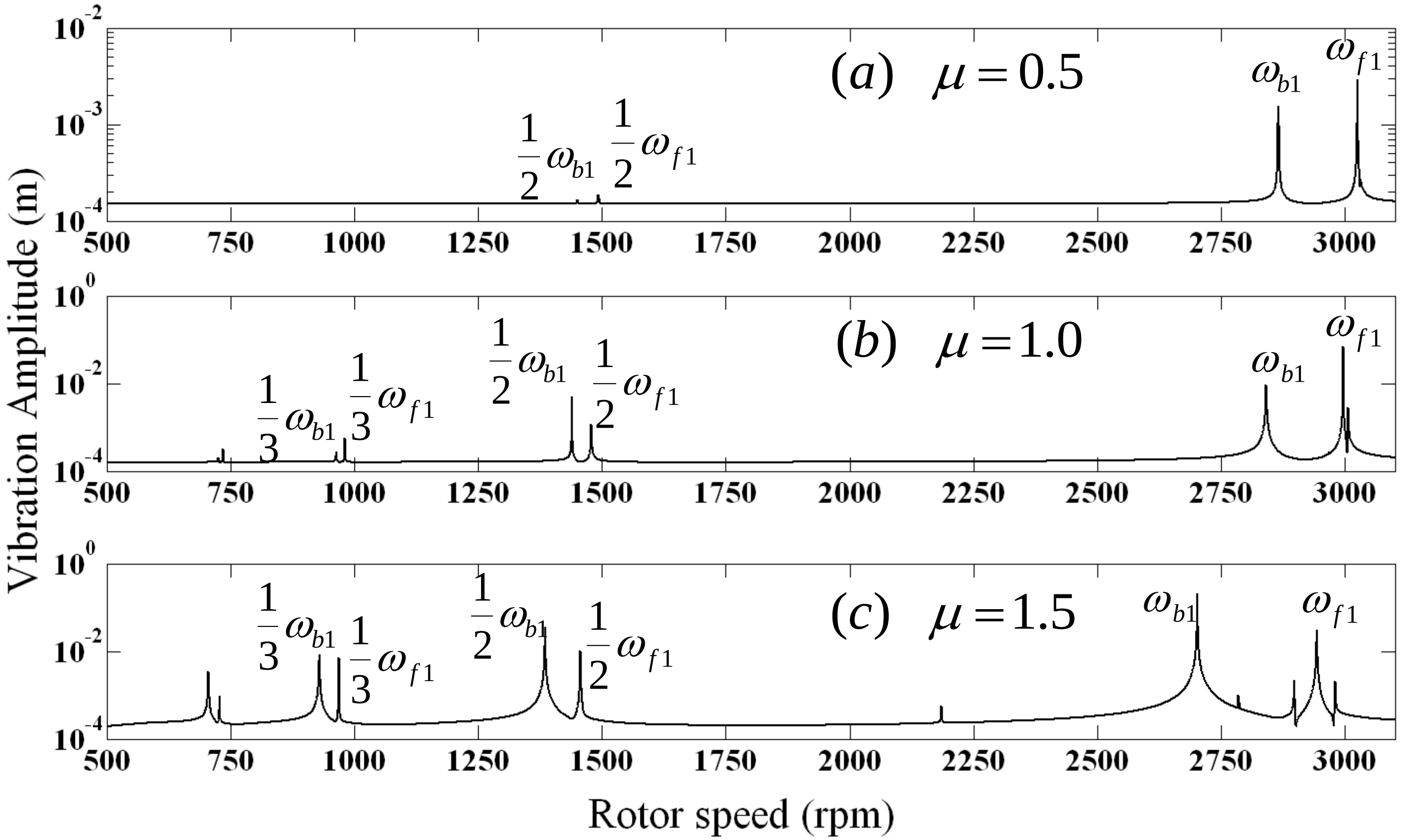


(a) Breathing crack model



(b) Open crack model

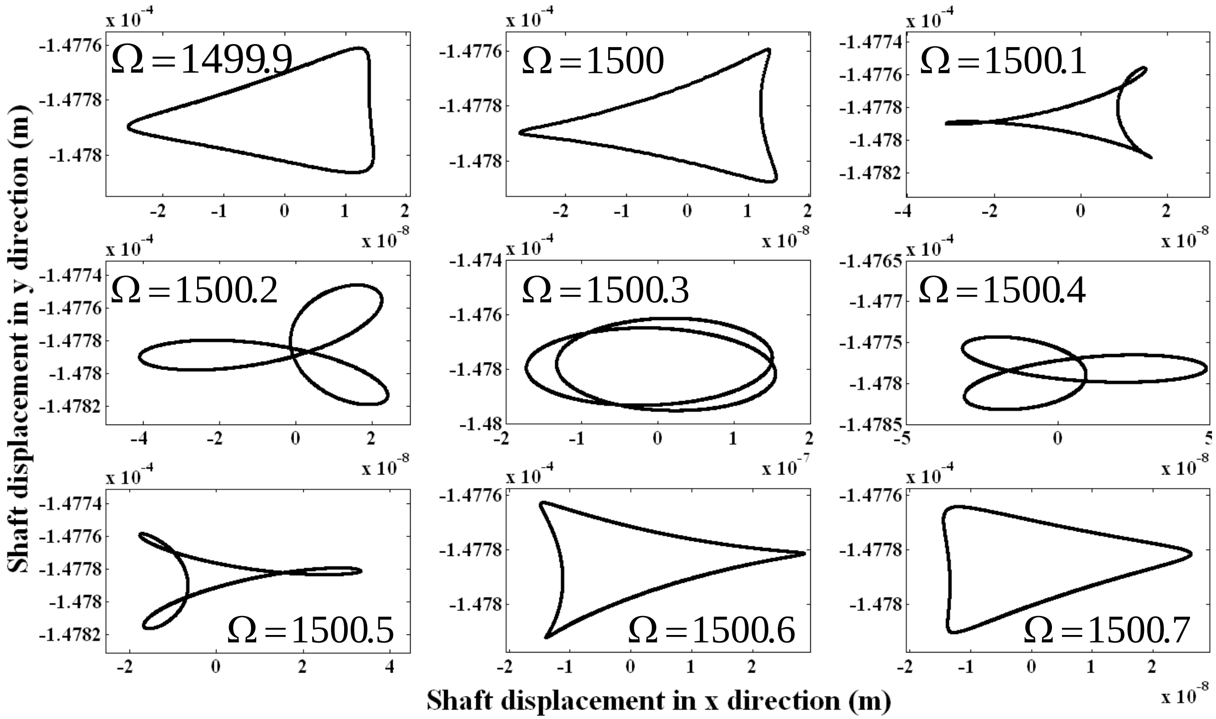
Damage Detection in Rotor-Bearing-Disk system
Crack Model: Breathing Crack Model



The vibration amplitudes at node 10

Damage Detection in Rotor-Bearing-Disk system
Crack Model: Breathing Crack Model

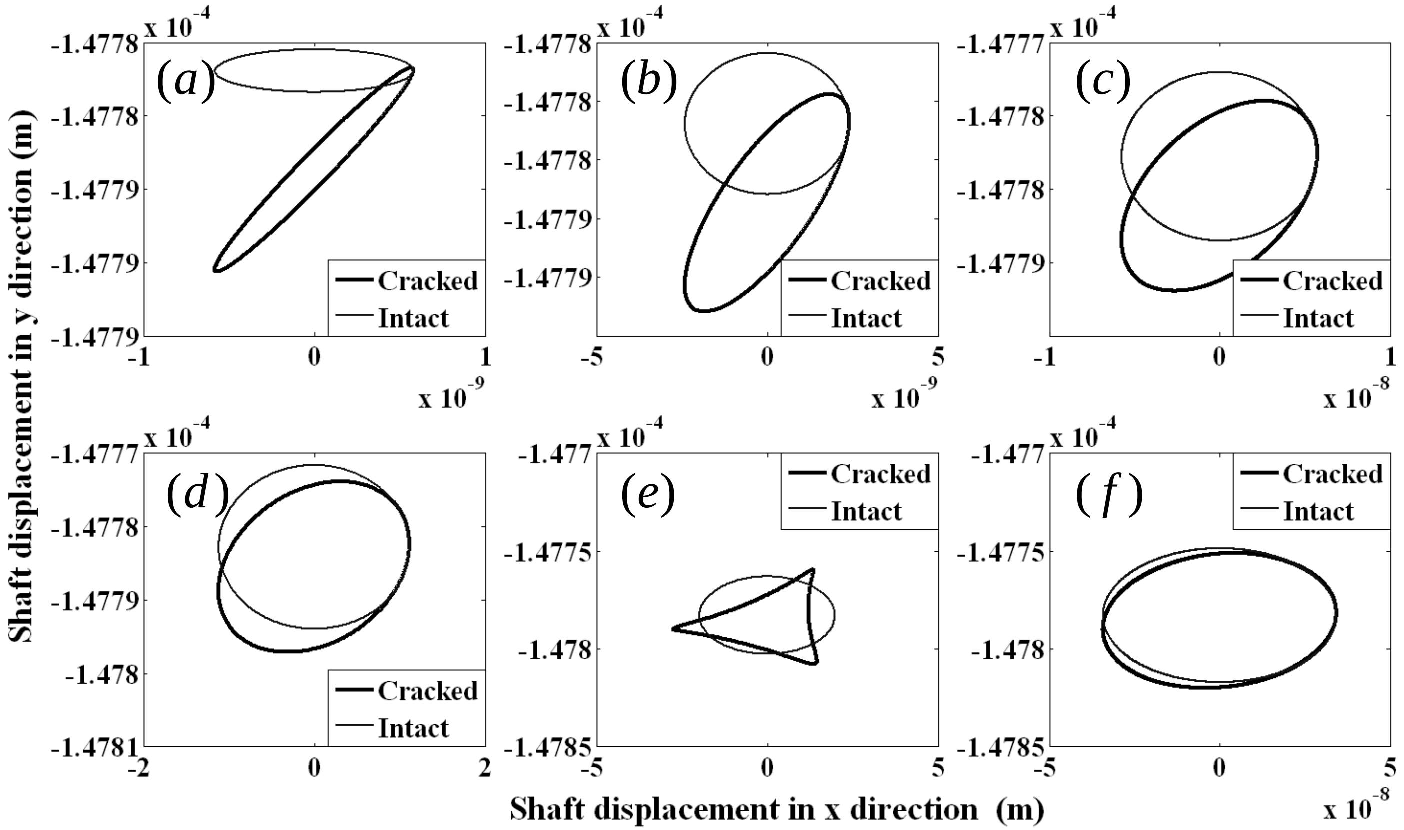
Shaft orbits of node 10 at the neighborhood of the sub-harmonic frequency $\Omega \cong (1/2)\omega_{f1} = 1500.3rpm$ at $\mu = 0.05$



$\Omega\,(rpm)$

Damage Detection in Rotor-Bearing-Disk system
Crack Model: Breathing Crack Model

Shaft orbits of node 10 for different shaft speeds at $\mu = 0.05$

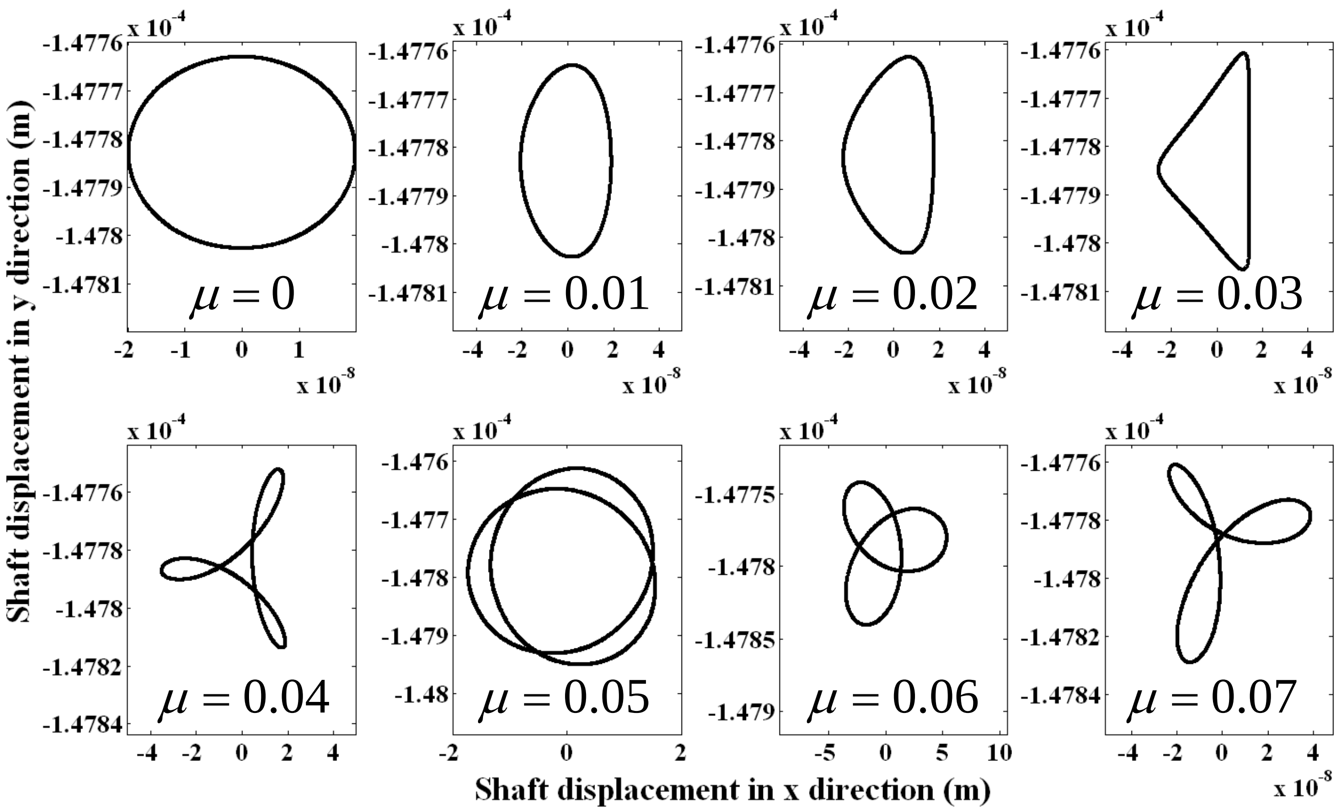


(a) $\Omega = 300rpm$ (b) $\Omega = 600rpm$ (c) $\Omega = 900rpm$

(d) $\Omega = 1200rpm$ (e) $\Omega = 1500rpm$ (f) $\Omega = 1800rpm$

Damage Detection in Rotor-Bearing-Disk system
Crack Model: Breathing Crack Model

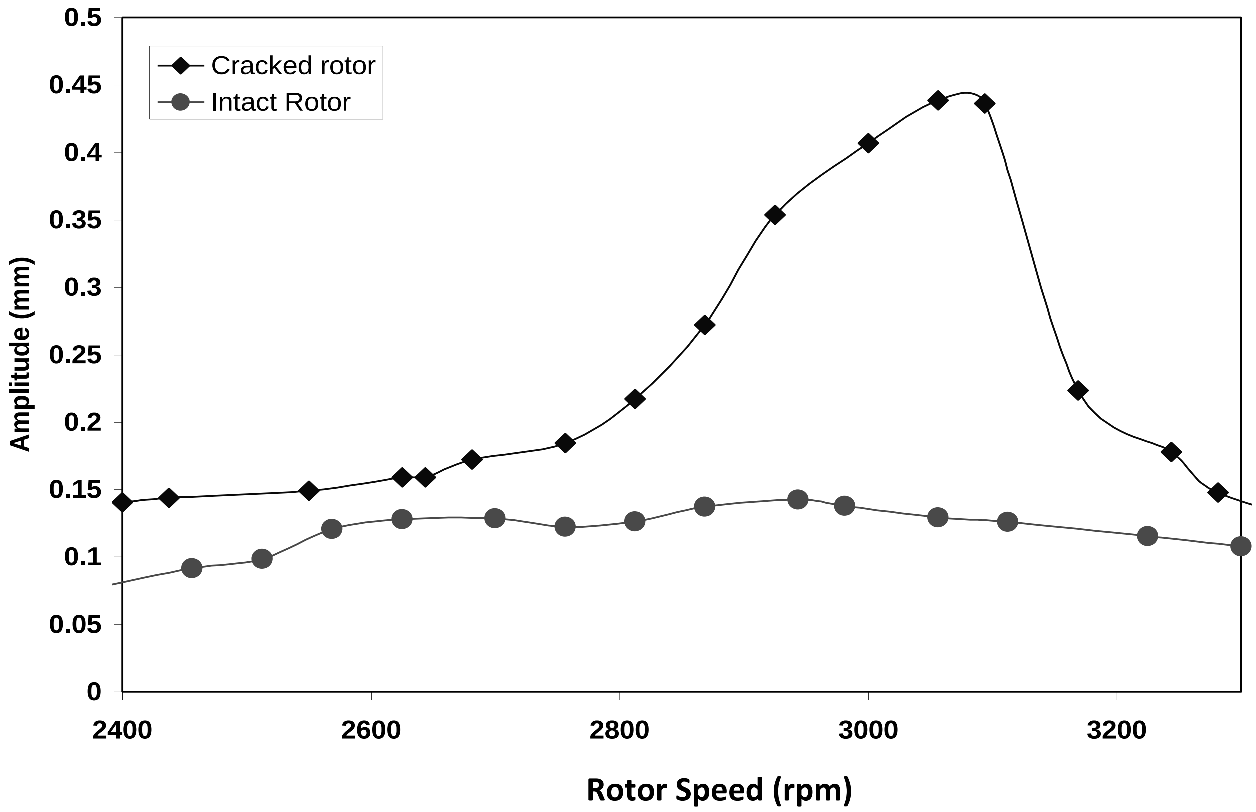
Shaft orbits of node 10



Shaft speed $\Omega = 1500.3rpm$

Damage Detection in Rotor-Bearing-Disk system
Crack Model: Open Crack Model- Experimental

Experimental results



Crack location is at element 13, $\mu = 0.4576$