Vibration-Based Techniques in Structural Health Monitoring

(NASA EPSCoR Progress Report for 2009/10)

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Collaborators: Thomas D. Burton, Ivan Argatov, Michael Todd, Gyuhae Park

Major Accomplishments of Past Year

- Methods to separate change in natural vibration frequencies due to internal damage to changes due to boundary damage in structures with bolted joints

Argatov, I.I. and E.A. Butcher, "On the Separation of Boundary and Internal Damage in Beam Structures with Bolted Joints," *Journal of Sound and Vibration*, submitted.

Argatov, I.I. and E.A. Butcher, "On the Iwan Models for Lap-Type Bolted Joints," *International Journal of Nonlinear Mechanics*, submitted.

- Method to obtain natural frequencies of beams & plates with damaged boundaries using Chebyshev polynomials

Sari, M. and E.A. Butcher, "Natural Frequencies and Critical Loads of Beams and Columns with Damaged Boundaries using Chebyshev Polynomials," *International Journal of Engineering Science*, accepted.

Sari, M. and E.A. Butcher, "Free Vibration Analysis of Kirchoff Plates with Damaged Boundaries by the Chebyshev Collocation Method," Symposium on Mechanics of Slender Structures (MOSS 2010), Donostia – San Sebastian, Spain, July 21-23, 2010.

- New computational techniques for order reduction of nonlinear systems

Al-Shudeifat, M. A., E. A. Butcher, and T. D. Burton, "Enhanced Order Reduction of Forced Nonlinear Systems using New Ritz Vectors," Proc. 28th International Modal Analysis Conference, Jacksonville, FL, Feb. 1-4, 2010.

Al-Shudeifat, M.A. and E.A. Butcher, "Enhanced Order Reduction Techniques of Forced Nonlinear Systems using New Types of Ritz Vectors," *Nonlinear Dynamics*, submitted.

Butcher, E. A., Al-Shudeifat, M. A., "An Efficient Mode-Based Alternative to Principal Orthogonal Modes in the Order Reduction of Structural Dynamic Systems with Grounded Nonlinearities," *Mechanical Systems and Signal Processing*, submitted.

Al-Shudeifat, M. and E. A. Butcher, "On the Dynamics of a Beam with Switching Crack and Damaged Boundaries: Application of the Local Equivalent Linear Stiffness Method," 13th Conference on Nonlinear Vibrations, Dynamics, and Multibody Systems, VPI&SU, Blacksburg, VA, May 23-27, 2010.

- New harmonic balance technique for identifying effects of breathing and open cracks in rotating shafts to critical and sub-critical speeds and experimental verification with a SpectraQuest rotordynamic simulator

Al-Shudeifat, M. and E. A. Butcher, "Identification of the Critical Crack Depths and Locations of Rotordynamic Systems in Backward Whirl," 7th International Workshop on Structural Health Monitoring, Sep. 7-11, 2009, Stanford, CA.

Al-Shudeifat, M., E.A. Butcher, and C. Stern, "Harmonic Balance Solution of a Cracked Rotor-Bearing-Disk System for Harmonic and Sub-Harmonic Analysis with Experimental Verification," *International Journal of Engineering Science*, accepted.

Al-Shudeifat, M.A. and E.A. Butcher, "New Breathing Functions for the Transverse Breathing Crack of the Cracked Rotor System: Approach for Critical and Subcritical Harmonic Analysis," *Journal of Sound and Vibration*, submitted.

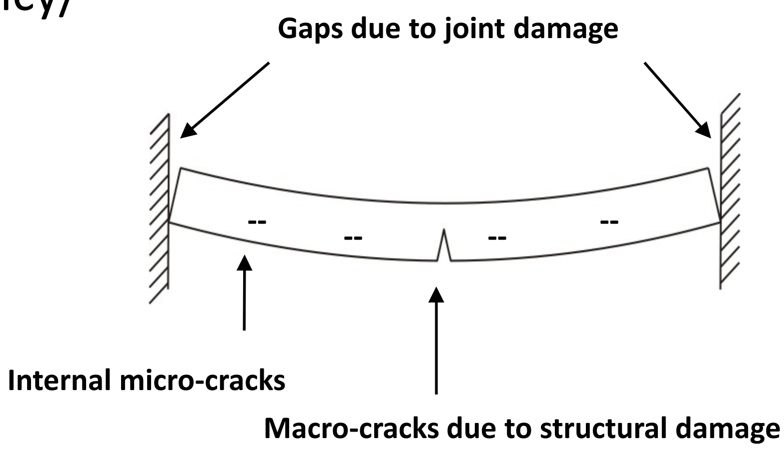
Al-Shudeifat, M. and E. A. Butcher, "On the Modeling of Open and Breathing Cracks of a Cracked Rotor System," proceedings of 2010 ASME IDETC, Montreal, Quebec, Aug. 15-18, 2010, accepted.

- Experimental verfication of hyperchaotic interrogation with attractor-based metric

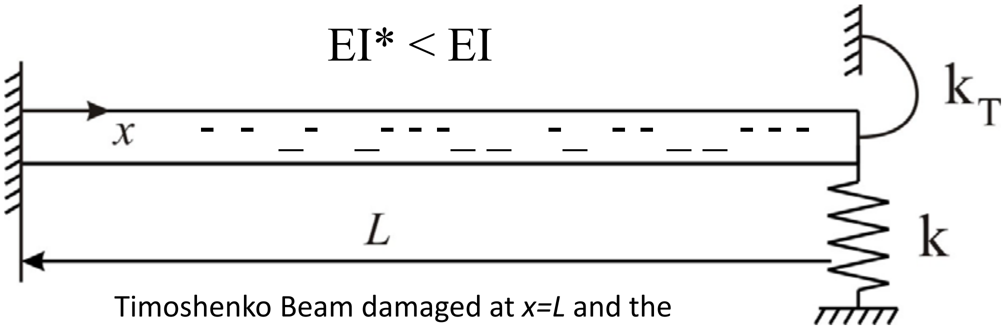
Torkamani, S., Butcher, E.A., Todd, M.D., Park, G.P., "Damage Assessment Using Hyperchaotic Excitation and State Space Geometry Changes," 2010 Inverse Problems Symposium, June 6-8, 2010, East Lansing, MI, accepted.

Types of structural damage

- Internal micro-cracks change elastic properties (global vs. local)
- Internal macro-cracks change local elastic properties, and hence the natural vibration frequencies and mode shapes
- Joint damage changes elastic properties at boundaries, frequency/ mode shapes, & adds damping
- Fundamental frequencies are lowered while adding higher harmonics



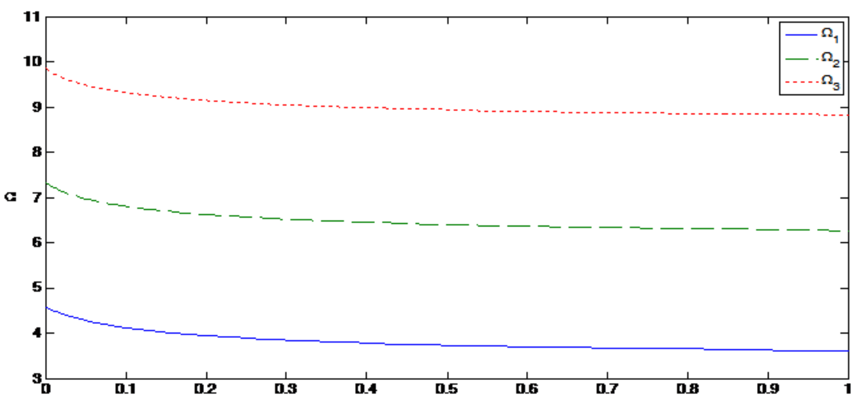
Obtaining Vibration Frequencies of Structures with both Internal (micro-) Damage and Boundary Damage



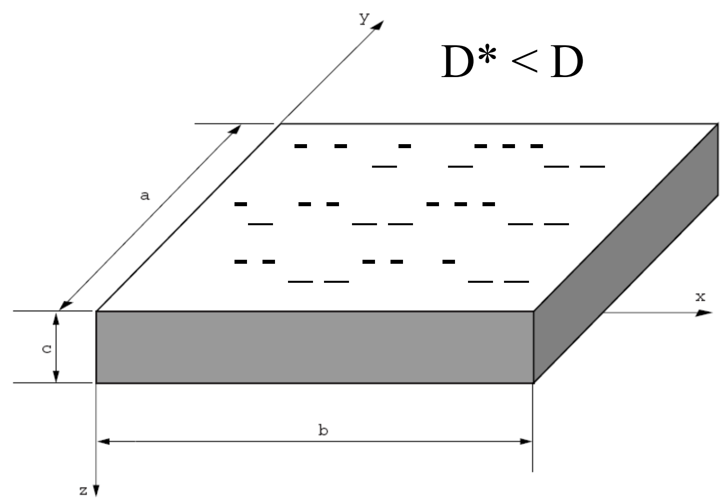
Timoshenko Beam damaged at $x=L$ and the other end clamped.

Nondimensional boundary damage parameters:

$$\alpha = \frac{EI^*}{kL^3} \qquad \alpha_T = \frac{EI^*}{k_T L}$$

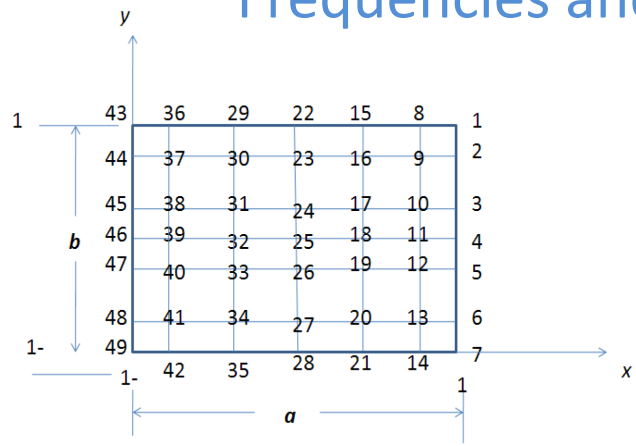


The first three natural frequencies versus the damage parameters, for a clamped-damaged Timoshenko beam, with h/L=0.1, ν=0.3, κ =5/6.

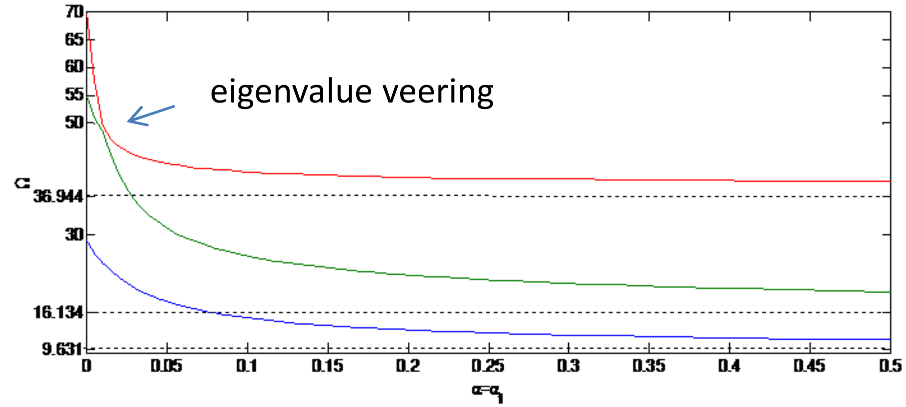


The *Chebyshev spectral collocation method* is implemented to determine 2-D and 3-D vibration frequencies for rectangular plates with damaged boundaries

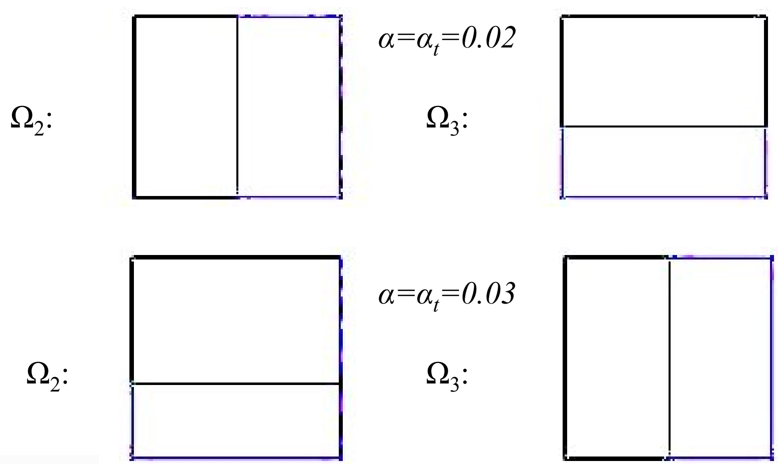
Damaged Kirchoff Plates: Effect of Damage on Free Vibration Frequencies and Eigenvalue Veering



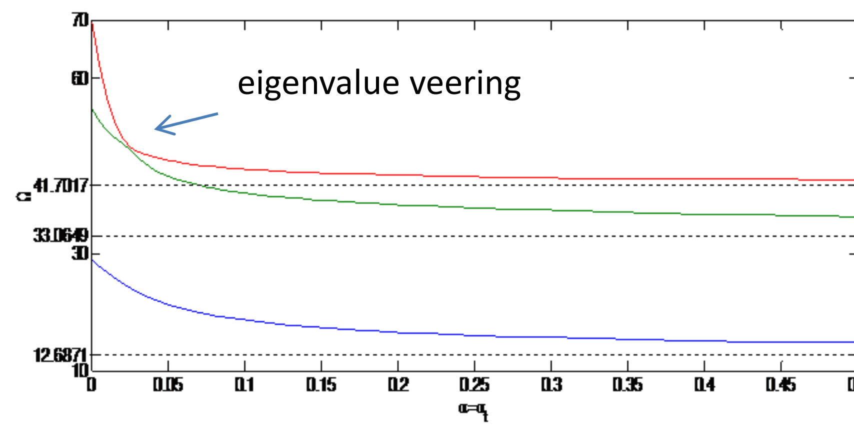
2D 7 X 7 Chebyshev grid used for the Kirchoff plate



The first three natural frequencies versus damage parameter for a SS-D-SS-D square Kirchoff plate.



The second and third mode shapes before and after the eigenvalue veering of the SS-C-SS-D square plate



The first three natural frequencies versus damage parameter for a SS-C-SS-D square Kirchoff plate.

Order Reduction of Large-Scale Dynamic Systems with Isolated Nonlinearities

Relevance to structural health monitoring :

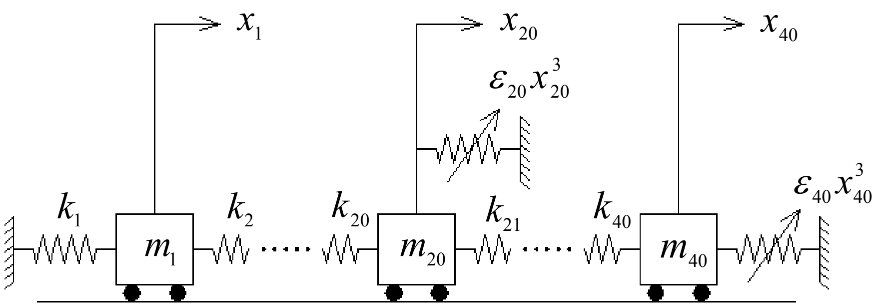
1. Small instruments using for measurement/ data collection
2. Allows a focus on a limited portion of the frequency spectrum
3. Model updating due to damage
4. Controlling a damaged structure (vibration localization)

Strategies:

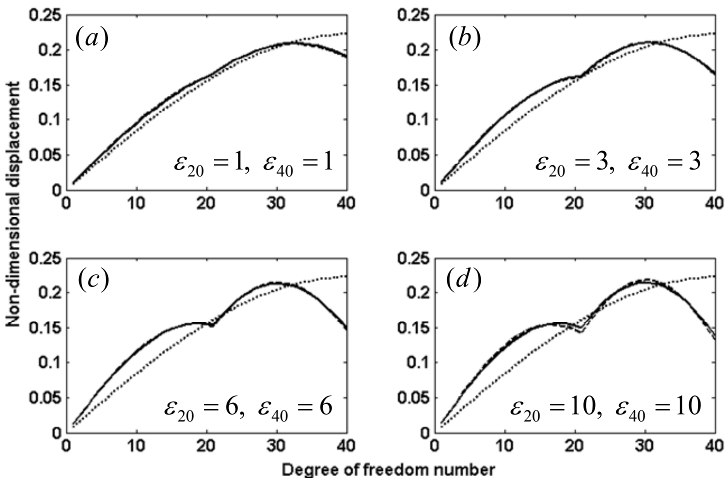
1. Utilize analytical technique (LELSM) which yields linear modes comparable in accuracy to Principal Orthogonal Modes
2. Utilize new forms of Ritz vectors at locations corresponding to isolated nonlinearities and forcing

Example : 40-DOF Mass-Spring System with Two Cubic Nonlinearities

$40-dof$ mass-spring system, $f(x_{40}) = \varepsilon_{40} x_{40}^3$ and $f(x_{20}) = \varepsilon_{20} x_{20}^3$

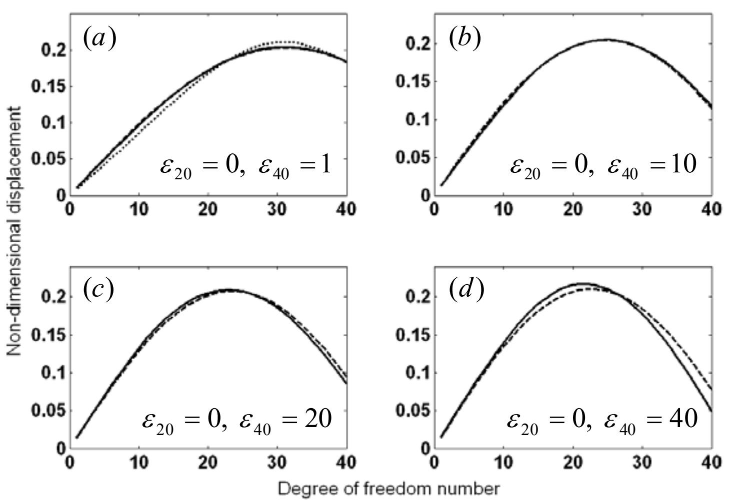


Two cubic springs



First POM (solid), first iterated LELSM mode (6 iterations-dashed)
Initial condition (1st linear mode-dots)

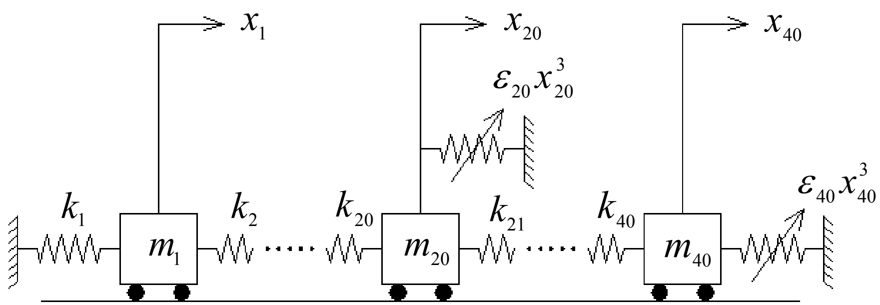
One cubic spring



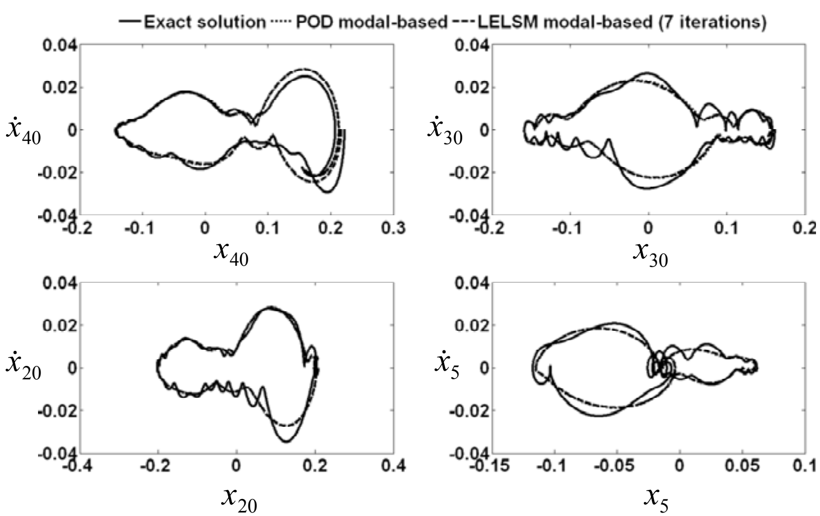
First POM (solid), first iterated LELSM mode (6 iterations-dashed)
1st NNM mode (dots)

Example : 40-DOF Mass-Spring System with Two Cubic Nonlinearities

$40-dof$ mass-spring system, $f(x_{40})=\varepsilon_{40}x_{40}^3$ and $f(x_{20})=\varepsilon_{20}x_{20}^3$



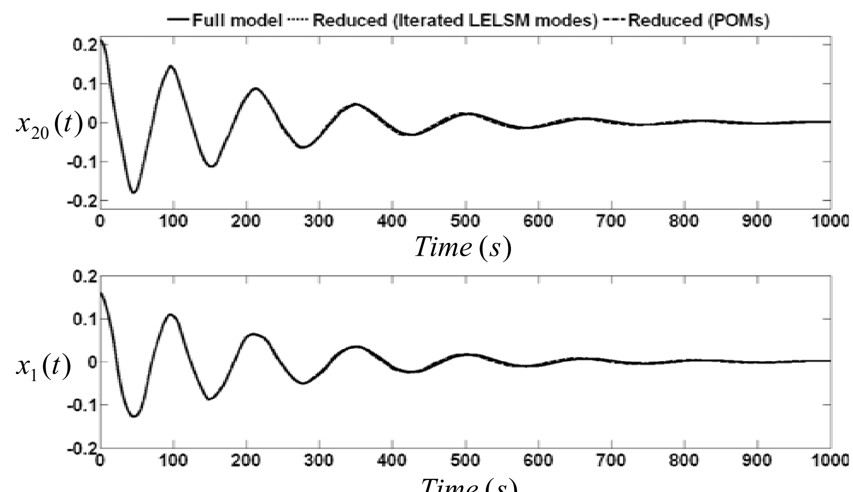
Undamped System



$\varepsilon_{20}=3,\ \varepsilon_{40}=3.\ \mathrm{x}(0)=\phi_1^{Linear},\ \dot{\mathrm{x}}(0)=0$

Both LELSM and POD reduced models use 10 modes

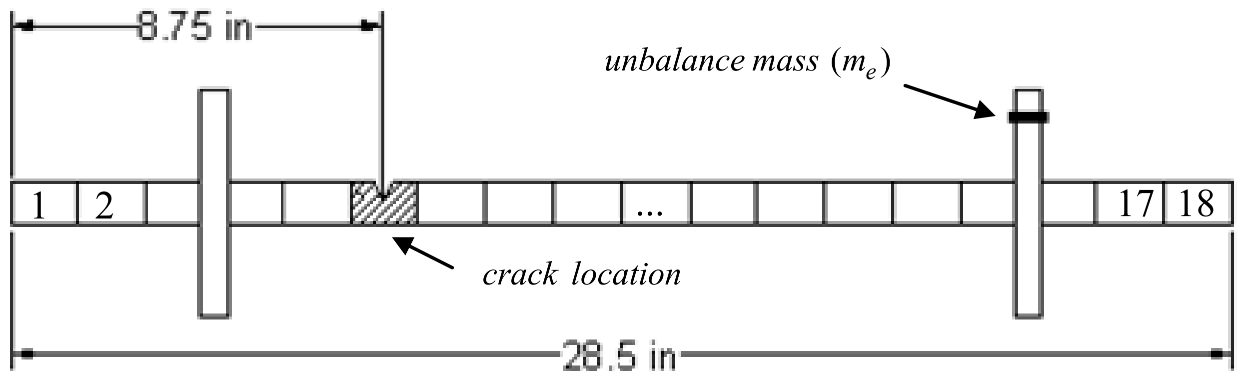
Damped System



$\varepsilon_{20}=3,\ \varepsilon_{40}=3.\ \mathrm{x}(0)=\phi_1^{LELSM},\ \dot{\mathrm{x}}(0)=0$

Both LELSM and POD reduced models use 10 modes

Damage Detection in Rotor-Bearing-Disk System
Breathing Crack Model

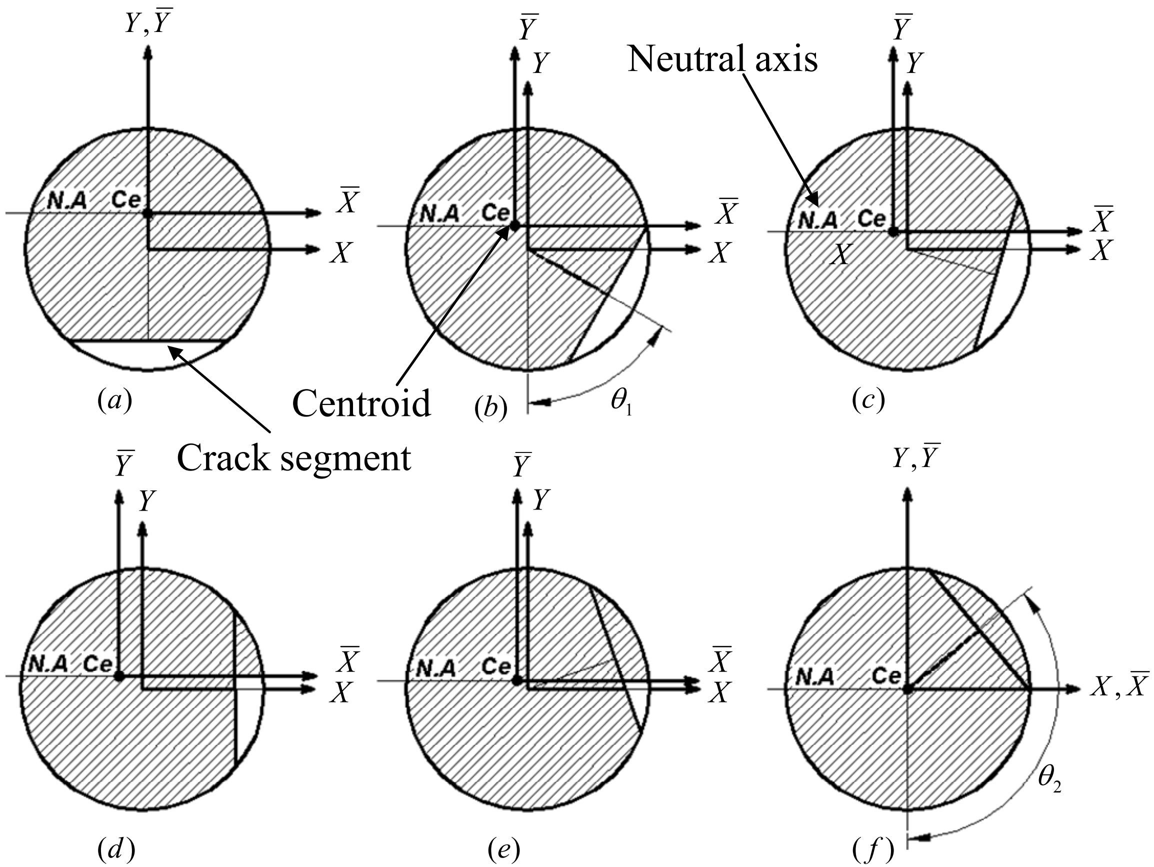


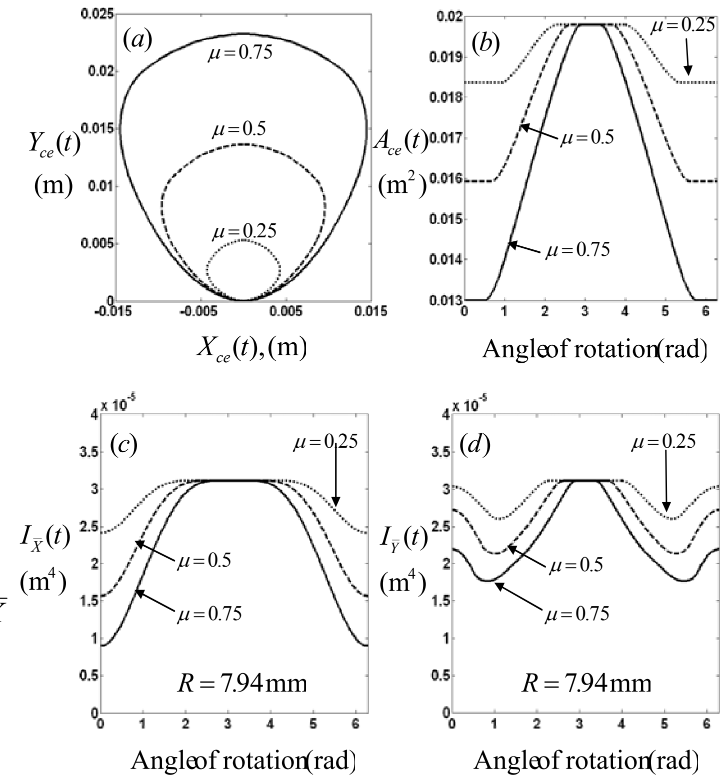
Finite element model of rotor-disk-bearing system

TABLE I PHYSICAL PARAMETERS OF THE MFS-RDS ROTOR-DYNAMIC SIMULATOR

Description	Value	Description	Value
Length of the rotor, L	$28.5\ in$	Disk outer radius, R_o	$3\ in$
Radius of the rotor, R	$0.625\ in$	Disk Inner radius, R_i	$0.625\ in$
Density of rotor, ρ	$7800\ kg/m^3$	Density of disk, ρ	$2700\ kg/m^3$
Modulus of elasticity, E	$2.1\times10^{11}\ N/m^2$	Mass of the disk, m_d	$0.571\ kg$
Bearing stiffness, (k_{xx}, k_{yy})	$7\times10^7\ N/m$	Mass unbalance, $m_e d$	$10^{-7}\ kg.m$
Bearing damping, (c_{xx}, c_{yy})	$5\times10^2\ Ns/m$	Mass unbalance angle, α	$0\ rad$

Actual Breathing Mechanism for the Breathing Crack Model





Time-Varying Centroidal Area Moments of Inertia of the Cracked Element Cross-Section

Actual Breathing Crack Model
Two New Breathing Functions

$$I_{\bar{X}} \cong \hat{I}_{\bar{X}}(t) = I - \left(I - \bar{I}_1\right) f_1(t) \quad \text{(A)}$$

$$f_1(t) = \left(\cos\left(\frac{1}{2}\Omega t\right)\right)^m = \frac{1}{2^m}\left(\binom{m}{m/2} + 2\sum_{j=0}^{(m/2)-1}\binom{m}{j}\cos\left((m-2j)\frac{\Omega}{2}t\right)\right) \quad \text{(B)}$$

$$I_{\bar{X}}(t) + I_{\bar{Y}}(t) \cong \hat{I}_{\bar{X}}(t) + \hat{I}_{\bar{Y}}(t) = 2I - \left(2I - \bar{I}_1 - \bar{I}_2\right) f_2(t) \quad \text{(C)}$$

$$f_2(t) = \frac{1}{\pi}\left(-\frac{\theta_1 + \theta_2}{2} + \frac{2}{(\theta_2 - \theta_1)}\sum_{i=1}^{p}\frac{\cos(i\theta_2) - \cos(i\theta_1)}{i^2}\cos(i\Omega t)\right) \quad \text{(D)}$$

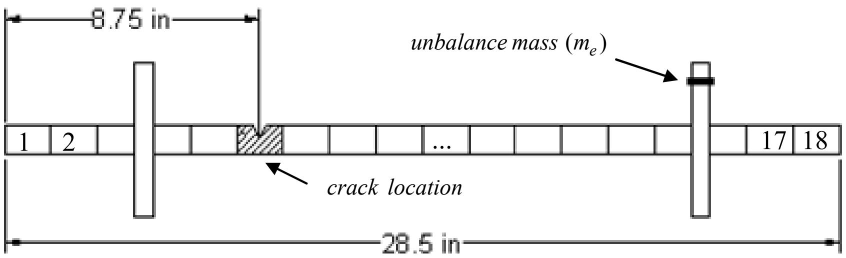
$$\bar{I}_1 = I_1 - A_1 e^2, \; \bar{I}_2 = I_2 \quad \text{(E)}$$

$$I_1 = I - I_x^c \;, \; I_2 = I - I_y^c \quad \text{(F)}$$

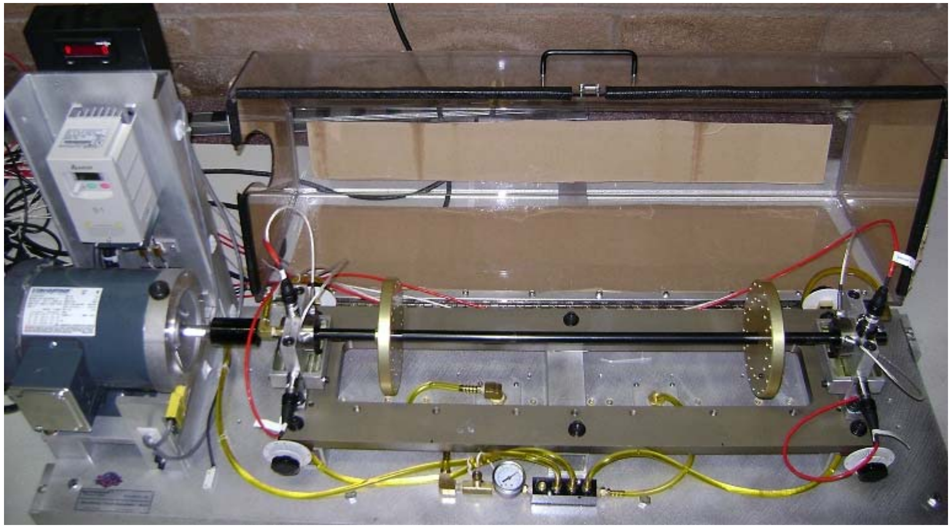
$$I_x^c = \frac{\pi R^4}{8} - \frac{R^4}{4}\left((1-\mu)(2\mu^2 - 4\mu + 1)\gamma + \sin^{-1}(1-\mu)\right) \quad \text{(G)}$$

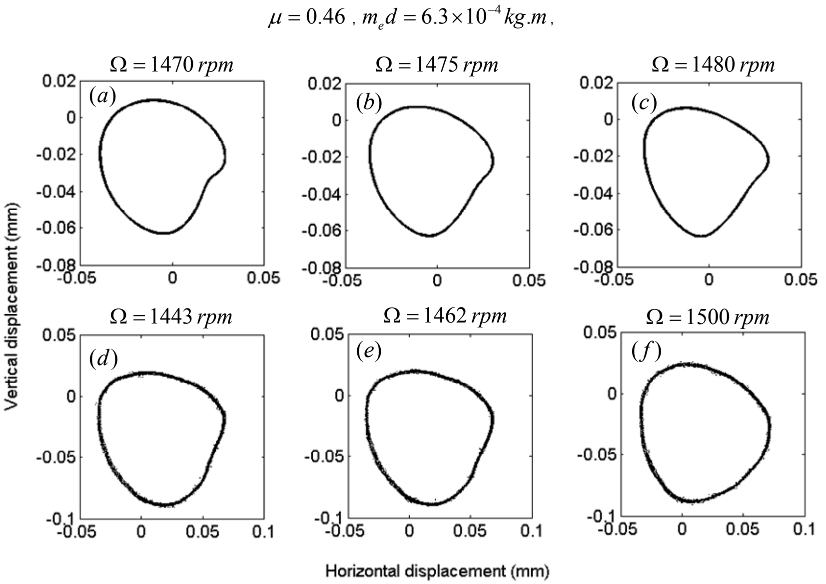
$$I_y^c = \frac{R^4}{12}\left((1-\mu)(2\mu^2 - 4\mu - 3)\gamma + 3\sin^{-1}(\gamma)\right) \quad \text{(H)}$$

Rotordynamic System with Breathing Crack in Shaft: FEA/HB Analysis vs. Experimental Verification



Finite element model of rotor-disk-bearing system



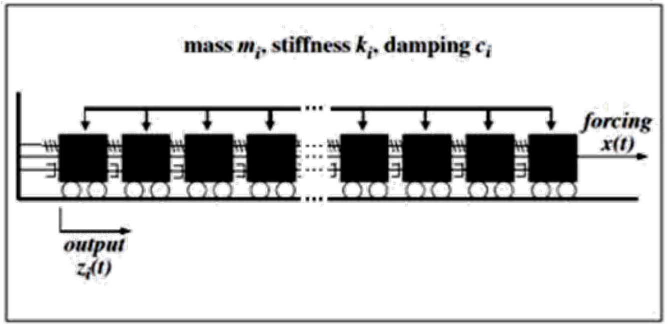


(a)-(c) Theoretical orbits, (d)-(f) Experimental orbits

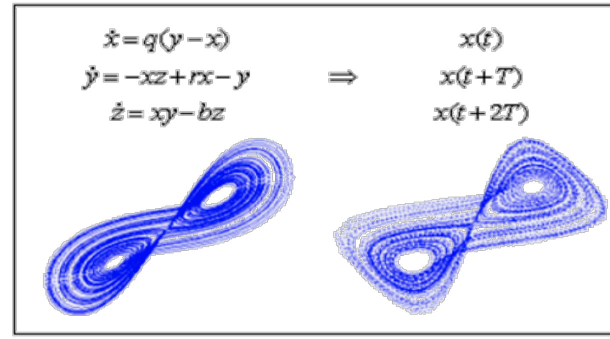
Comparison Between the Theoretical and Experimental Whirl Orbits
(in the Neighborhood of 1/2 of the First Critical Rotational Speed)

Damage Detection via Hyperchaotic Interrogation Using Attractor-Based Damage Metrics

8-DOF Structure (Filter)



Reconstructed Lorenz Hyperchaos



Chaotic Interrogation Technique [Todd *et al* 2001]

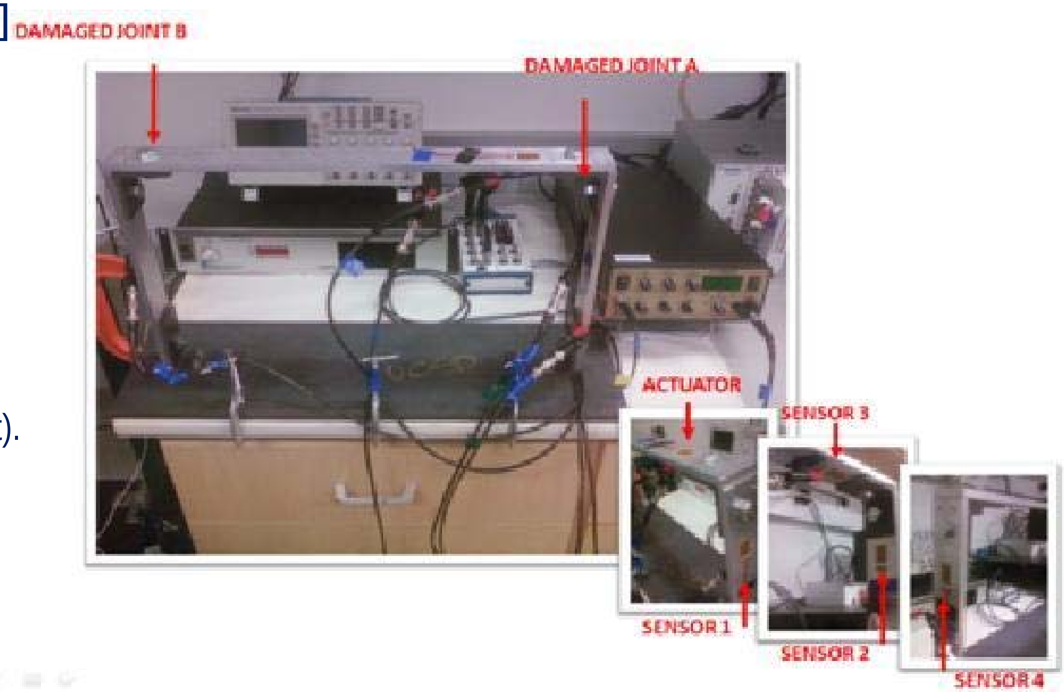
- Uses deterministic excitation
- Uses steady state response
- Uses attractor geometry as a feature

Chaotic signals are:

- Broadband in the frequency domain.
- Deterministic and low dimensional (as low as three-dimensional).
- Extremely sensitive to small changes in system parameters (having a positive Lyapunov exponent).

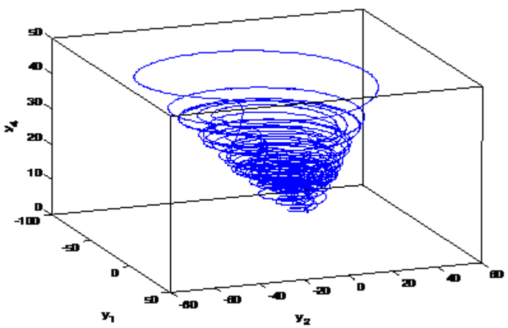
Why a Hyperchaotic excitation ?

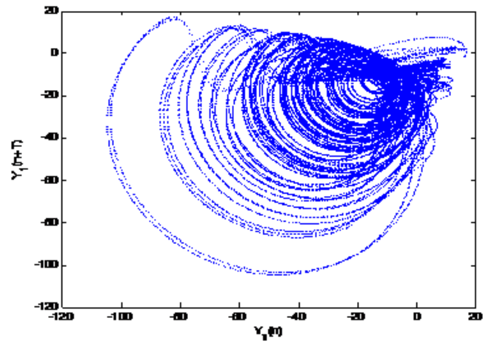
- More sensitive to small changes in system parameters due to having more than one positive Lyapunov exponent.
- Still low dimensional (as low as four-dimensional)



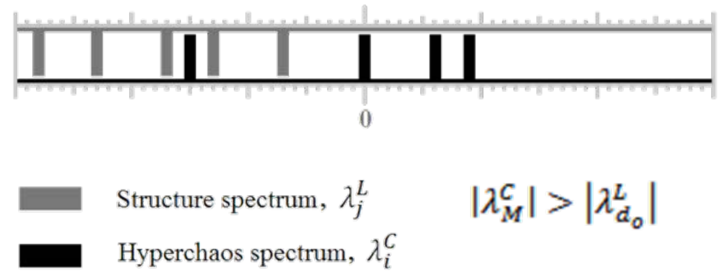
Damage Detection via Hyperchaotic Interrogation Using Attractor-Based Damage Metrics

Rossler Hyperchaos: Delay reconstruction





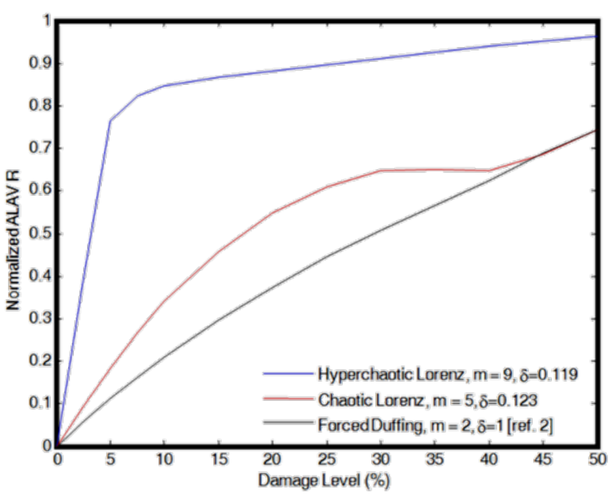
Tuning Criteria for Lyapunov Exponents of Hyperchaos and Filter



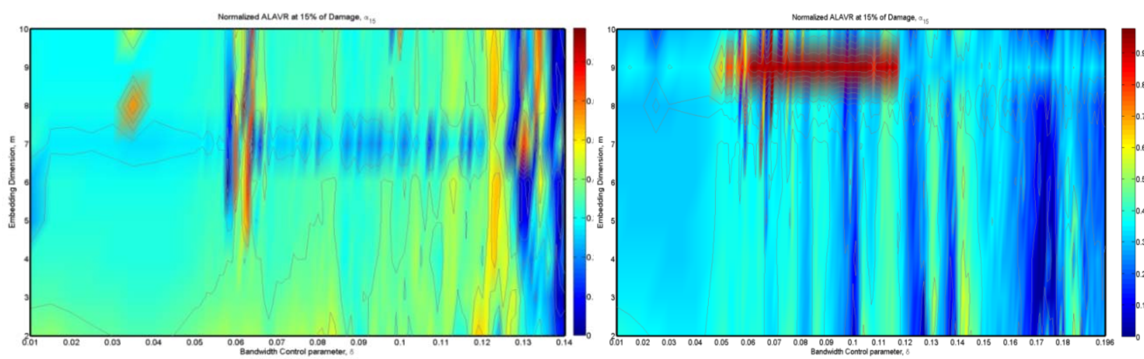
$$|\lambda_M^C| > |\lambda_{d_o}^L|$$

$$\sum_{r=1}^{d_o} |\lambda_r^L| > \sum_{r=1}^{p} |\lambda_r^C| > \sum_{r=1}^{d_o - 1} |\lambda_r^L|$$

Average Local Attractor Variance Ratio (ALAVR) as Damage Metric: Damage Sensitivity to Hyperchaotic (*more sensitive*) vs. Chaotic Interrogation (*less sensitive*)



Comparison of ALAVR Sensitivity to Hyperchaotic vs. Chaotic Interrogation at 15% Damage vs. Bandwidth/ Embedding Dim.



Current Work in Progress

- Apply method to separate internal and boundary damage to 3-D and Mindlin rectangular plates using Chebyshev collocation, and compare with results from a Perturbation Method (Butcher, Sari, and Nazari)

  Butcher, E.A., Sari, M., and Nazari, M., "Free Vibration Analysis of Kirchoff Plates with Damaged Boundaries by the Chebyshev Collocation and Perturbation Methods," accepted for ASME Conference on Smart Materials, Adaptive Structures, and Intelligent Systems, Sep. 28-Oct 1, 2010, Philadelphia, PA.

- Develop methodology for the analysis of natural frequencies of breathing cracks in cantilever beams with damaged boundaries (Butcher, Al-Shudeifat, Sari)

  Al-Shudeifat, M., Sari, M., and Butcher, E.A., "Modal Behavior of a Beam with Switching Crack and Damaged Boundaries: Application of the LELSM and Chebyshev Collocation Methods," accepted for ASME Conference on Smart Materials, Adaptive Structures, and Intelligent Systems, Sep. 28-Oct 1, 2010, Philadelphia, PA.

- Further develop method of hyperchaotic interrogation of structures with attractor-based damage metrics and verify in experiments (Butcher, Torkamani, with M. Todd and G. Park)

  Torkamani, S., Butcher, E.A., Todd, M.D., Park, G.P., "Damage Assessment Using Hyperchaotic Excitation and State Space Geometry Changes," accepted for ASME Conference on Smart Materials, Adaptive Structures, and Intelligent Systems, Sep. 28-Oct 1, 2010, Philadelphia, PA.

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