General Relativistic Effects in Galactic Rotation Velocity Profiles

Sophia Cisneros, NMSU, presenting `08 APS 4Corners

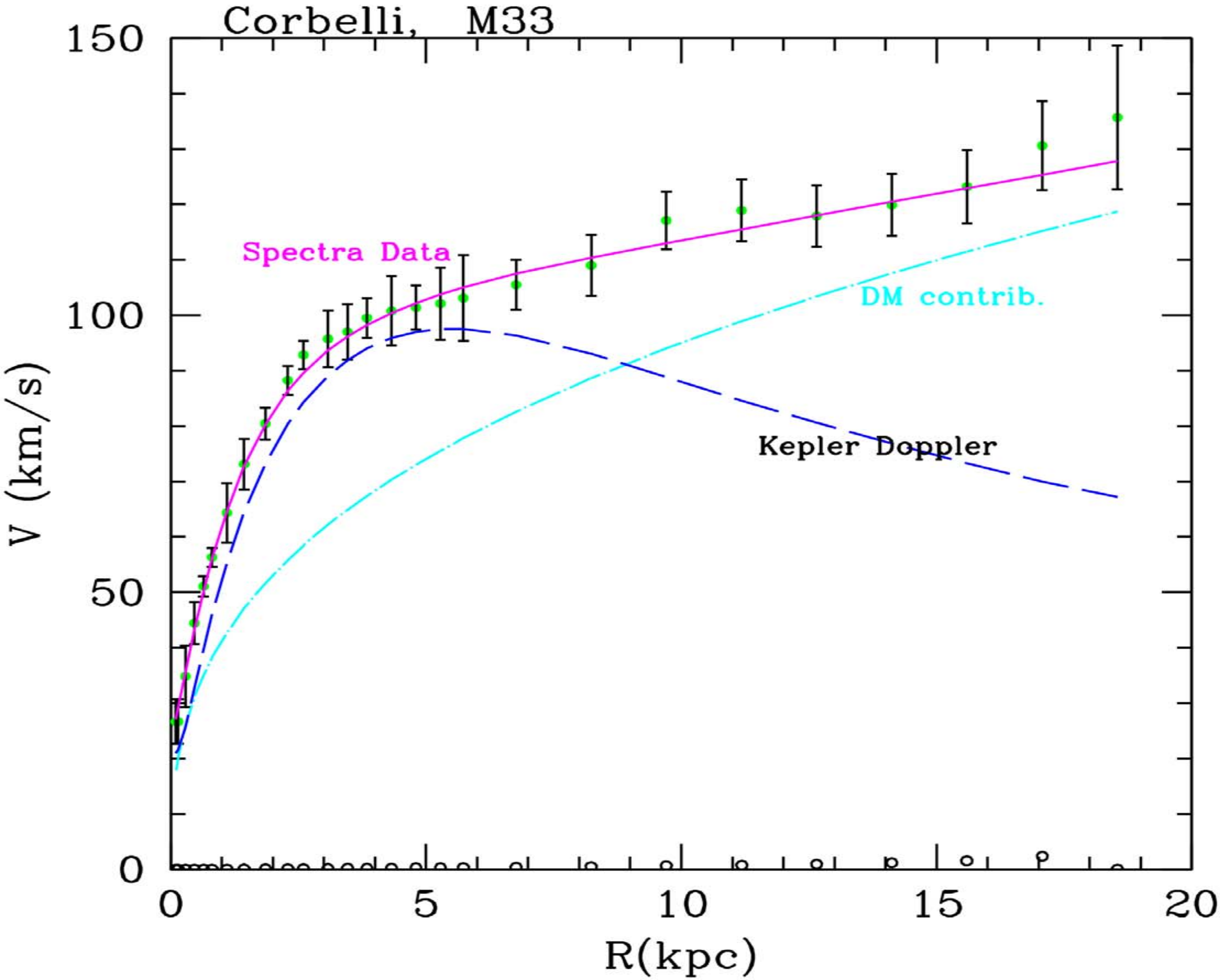
With support from::

NMSU

NASA

CCNY

The Dark Matter Problem



Corbelli, M33
150
100
50
0
V (km/s)
0
5
10
15
20
R(kpc)
Spectra Data
DM contrib.
Kepler Doppler

“a theory should not allow indefinite adhoc readjustments to take care of each new counter-result that may be found...”~Bohm

We posit, to be careful about what exactly is Dark Matter, we are required to ask;

- **What happens when a disc of matter, like a spiral galaxy,**
- **Spins in the vacuum?**

The clues. . .

- The phenomena of the URC----- curvature
- Wavelength dependence of velocities---dispersion

Translational doppler in flat space can be derived from the spacetime invariant, the inner product of the photon's four momentum and the emitting and receiving particles four velocity

$$p^{\alpha}u_{\alpha} = p^{\alpha}g_{\alpha\beta}u^{\beta} \tag{1}$$

returns

$$\omega_o = \omega\gamma(1 - \frac{v}{c}) \tag{2}$$

$$\gamma = u^t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

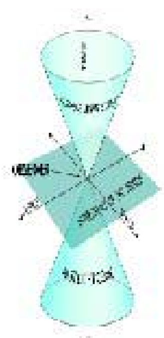


Fig. 1.—

But this implicitly assumes that the speed of light for the emitter and the observer are the same...

From Maxwell

- The wave equation tells us how the speed of propagation of an electromagnetic wave is changed in a medium different from a flat spacetime vacuum.....
- If c->vn   where the index of refraction is familiar from snell's law...

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = 0$$

We can solve for this relation in a curved space using the wave equation

(*thanks VP Nair)

$$\Box = \partial^\mu \partial_\mu = \eta^{\mu\nu} \partial_\nu \partial_\mu = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial t^2} \right) = \left(\nabla^2_{\mathbf{R}^3} - \frac{\partial^2}{\partial t^2} \right)$$

\but when the space curves, so does pointing of the time. WE do well

To picture the space covered with grasses-

the grass everywhere locally perpendicular to the surface, attached to the bending of space-

these are Our clocks-

The way time flows tells us about the Energy part of the Field Equations.....The energy part is the photon's frequency~

- A curved space- the clocks mix with the space

- Minkowski Flat Spacetime- the clocks are
- Everywhere orthogonal to
- The space...

This reasoning, for a Schwarzschild Metric reproduces the gravitational redshift

– 1 –

$$ds^2 = -(1 - 2M/r)dt^2 + \frac{1}{1 - 2M/r)}dr^2 + r^2 d\phi^2 \quad (1)$$

the D'Alembertian operator reduces to:

$$\Box = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\mu}(g^{\mu\nu}\sqrt{g}\frac{\partial}{\partial x^\nu}) \quad (2)$$

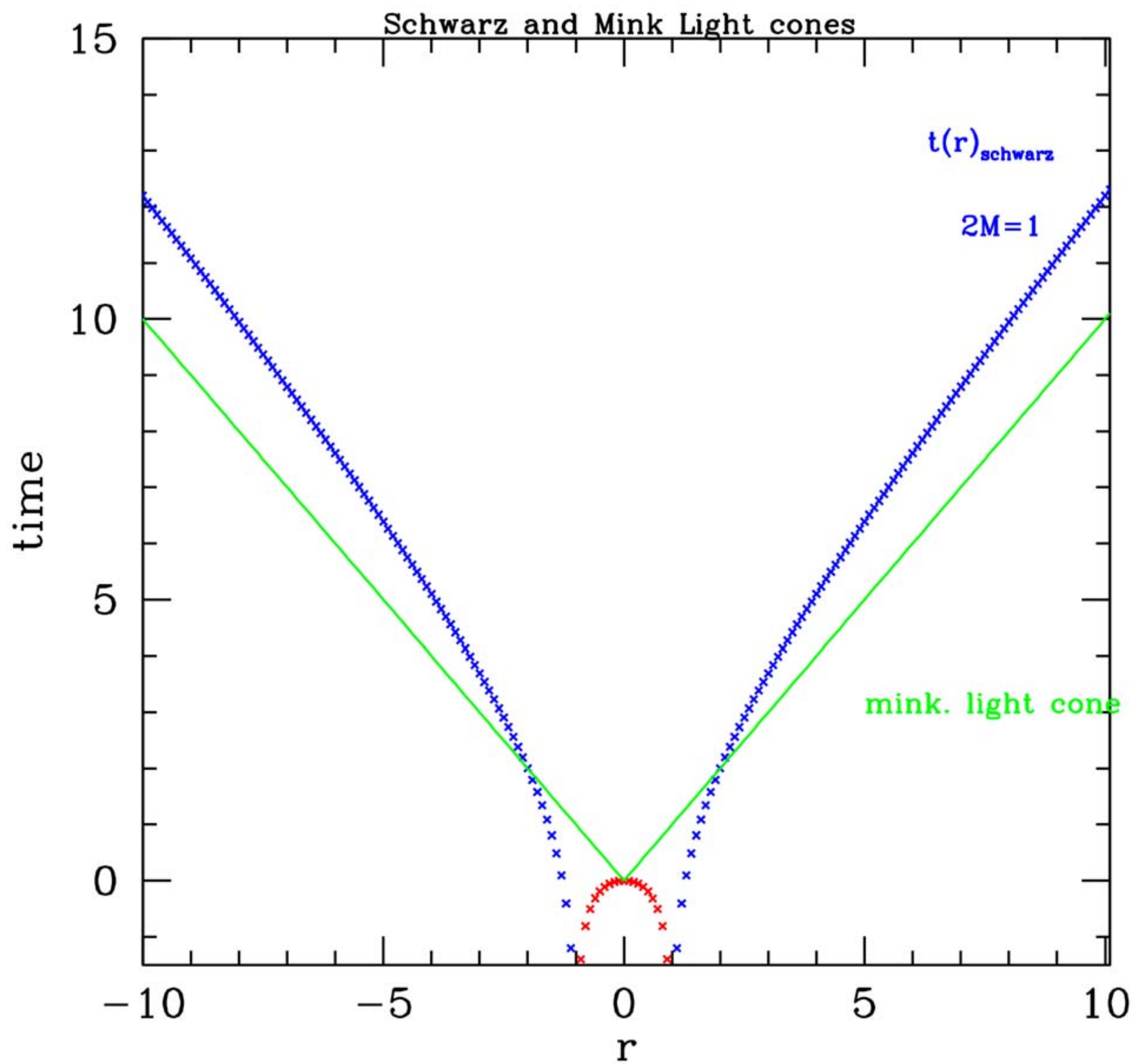
$$\Box = g^{tt}\frac{\partial^2}{\partial t} + g^{rr}\frac{\partial^2}{\partial r} + g^{\phi\phi}\frac{\partial^2}{\partial \phi} \quad (3)$$

We arrive at an effective index, relating the new frequency and wave number, ω and k, as

$$n = g^{00} = \frac{1}{g_{00}} = \frac{1}{1 - 2m/r} \quad (4)$$

Where:

$$n = c/v_{photon} = c\frac{k}{\omega} \quad (5)$$



Schwarz and Mink Light cones
15
10
5
0
time
t(r)schwarz
2M=1
mink. light cone
−10
−5
0
5
10
r

Kerr Wave Equation gives an effective index;

$ds^2 = -(1-d)dt^2 + \frac{1}{(1+b^2-d)}dr^2 + r^2(1+b^2(1+d))d\phi^2 - 2(rdb)d\phi dt$

where b is the angular momentum of the space, and d=2M/r. for equatorial orbits, the D'Alembertian operator becomes:

$$\Box = g^{tt}\frac{\partial^2}{\partial t} + g^{rr}\frac{\partial^2}{\partial r} + g^{\phi\phi}\frac{\partial^2}{\partial \phi} + g^{\phi t}\frac{\partial}{\partial \phi}\frac{\partial}{\partial t} \qquad (1)$$

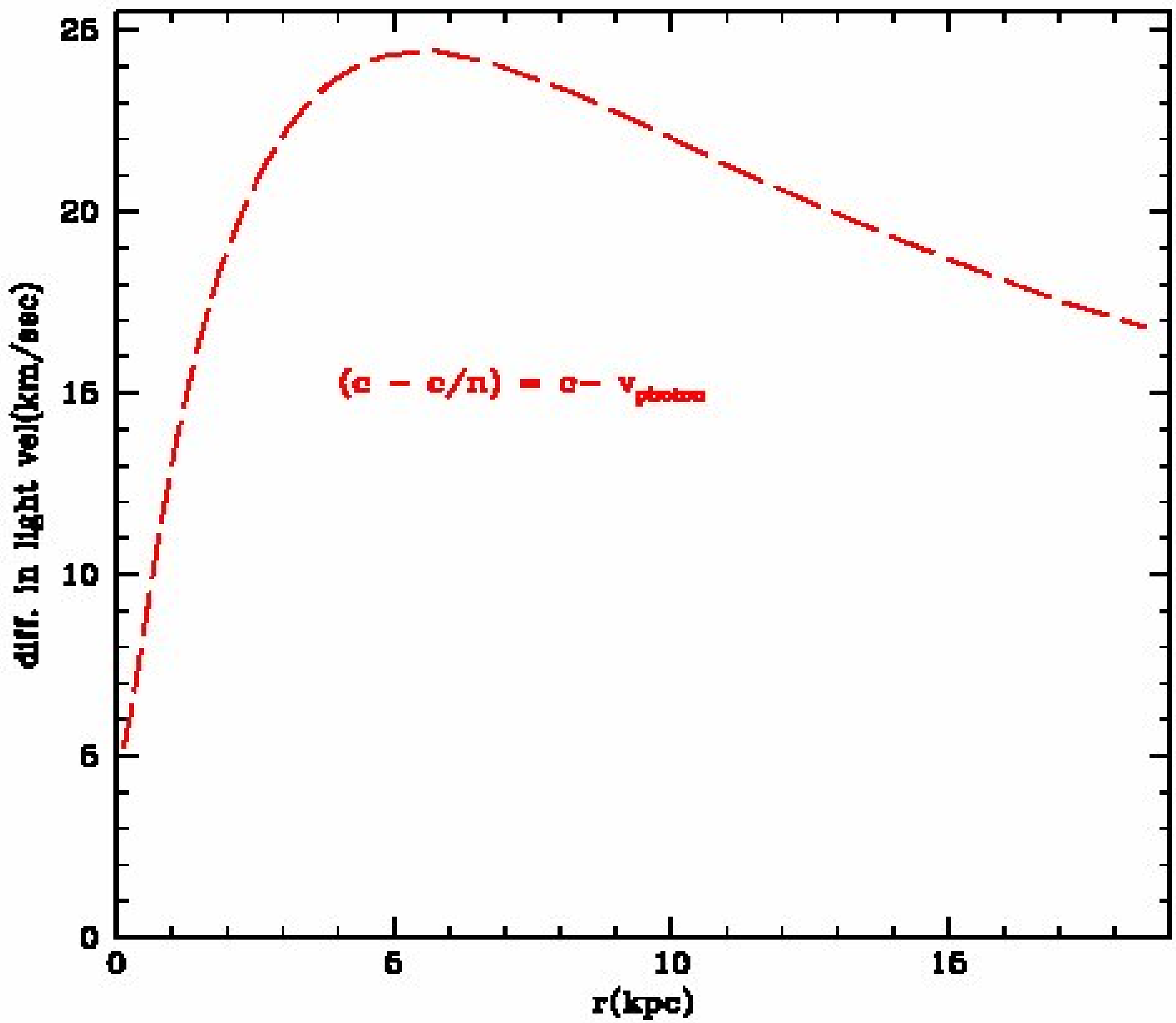
with, an effective index of::

$$n = \frac{\frac{r_o}{r}db + \sqrt{(\frac{r_o}{r}db)^2 + 4(1-2b^2+b^4(1-(\frac{r_o}{r})^2))(1+b^2(1+d)}}{2(1-2b^2+b^4(1-(\frac{r_o}{r})^2)} \qquad (2)$$

*where complex terms dropped as unimportant away from a resonance

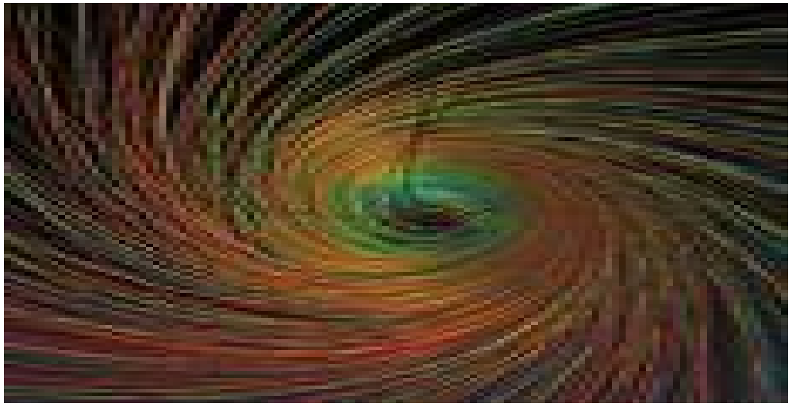
A Kerr light speed difference, M33

- As calculated for the implicit
- Assumption of Keplerian velocities within the disc



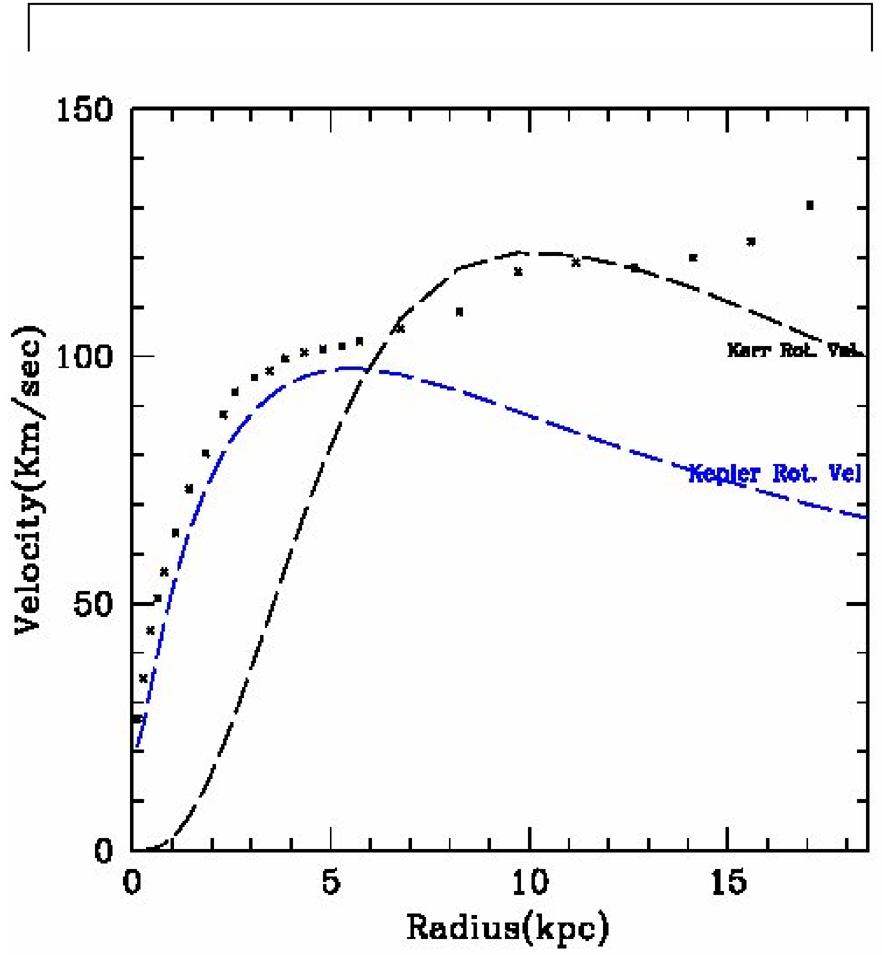
The disc drags the light with it! Similar to light moving through a fluid of variable density.

- current state of the art theory is done within the regime of the linearized Field Equations- and they find velocity contributions on the order of 10m/s.
- Here, the contributions of a Schwarzschild Metric alone would give 30m/s-
- here we avoid linearizing the equations by solving the wave equation in a Kerr Space! And then apply the invariant energy conditions!



Future work

- Use the Euler Lagrange equations, the Relatvst. Action
- To solve for the Kerr velocity
-



Acknowledgements

- Mottola, Emil- LANL
- Nair, VP- CCNY
- Engelhardt, Michael-NMSU
- Beetle, Chris-FAU
- Kyle, Gary-NMSU
- Walterbos, Rene-NMSU
- Corbelli, Edvige- ITY