Circle Packing Approach to Modeling van der Waals Forces

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- Introduction
  - Microassembly.
  - Adhesive Forces.
  - Van der Waals Forces
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- Geometric modeling of a cylindrical part using Circle Packing
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Introduction

- Microassembly
  - Process of manipulating micro-components (10^{-6}m) to build a micro-structures.
- Adhesive Forces
  - Forces that at the micro level cause “sticking” of components to manipulators.
  - Surface Tension, Electrostatic, and van der Waals [3][4].
- Van der Waals (vdW)
  - Adhesive force caused by a induced dipole polarization between atoms, and molecules [2].

Introduction

- Van der Waals (vdW).
  - Closed form solutions of vdW interactions between different geometries are ideal and limited.

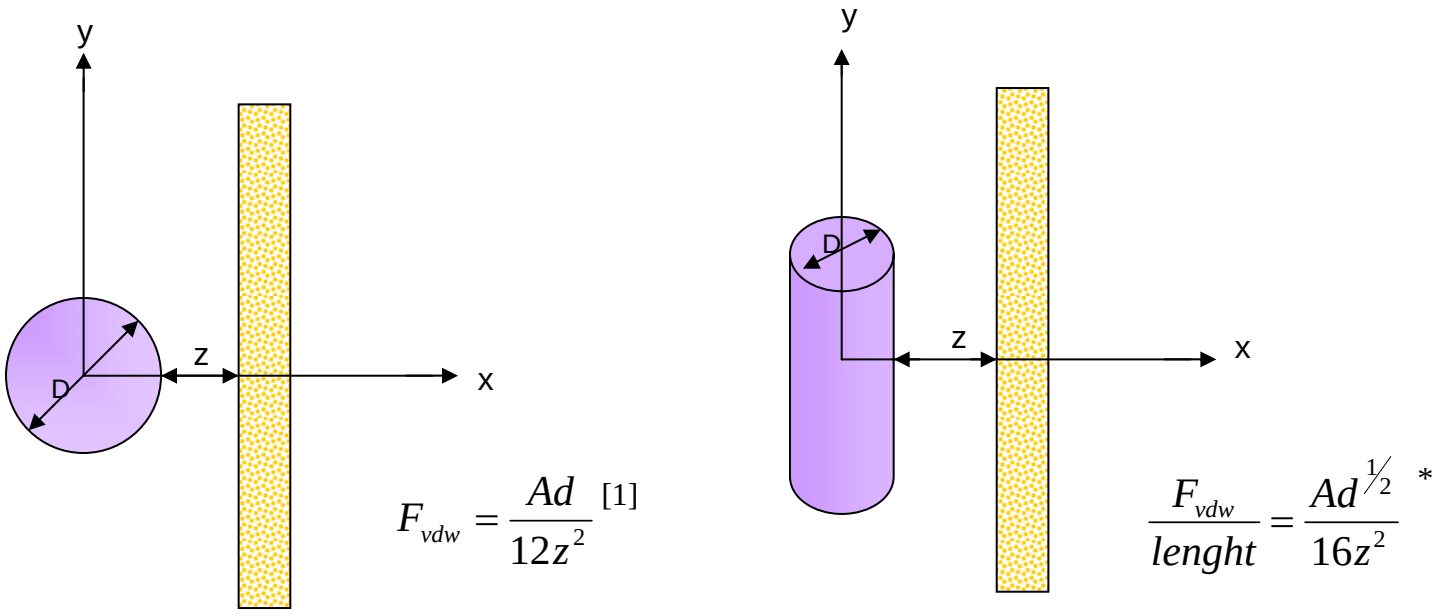


Figure I. Van der Waals force interaction between a sphere and a flat plane, and a cylinder and a flat plate. *A* is the Hamaker constant. *www.clarkson.edu/projects/fluidflow/courses/me537/5_vanderWaals.pdf

Research Objective

- Develop a finite method of modeling van vdW forces.
  - Depart from closed form solutions, that only model ideal shapes.
  - Model geometries of greater complexity.
  - Initial model, cylinder-flat plane interaction.
  - Use closed form solution sphere-plane as finite element.
- Why? To predict and plan for vdW interactions during assembly planning.

Geometric Modeling of a Cylindrical Part using Circle Packing

- 1. Circle Packing- mathematical modeling of packing circles into circles.
  - Provides a cross sectional template for cylinder.
  - Defines the coordinates (x_i, y_i) of individual circles i .

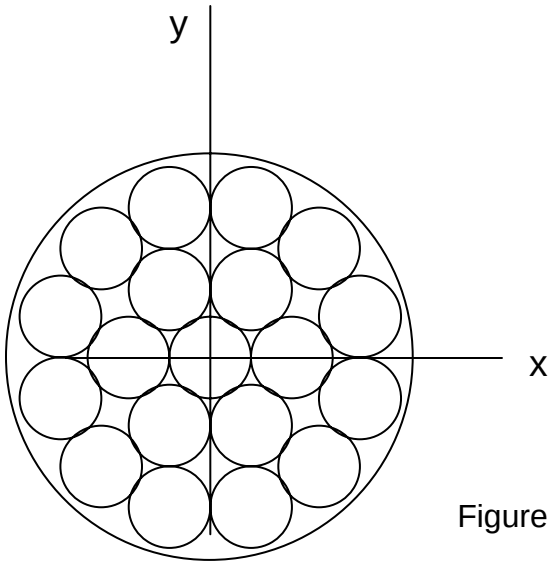
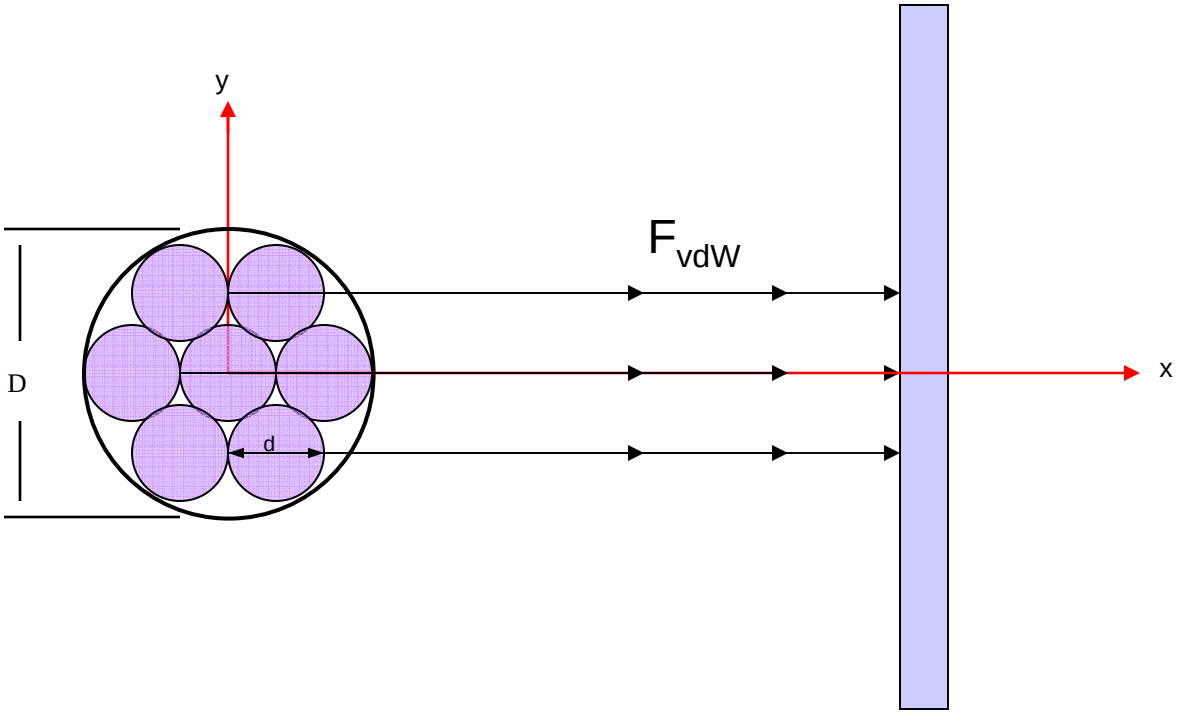


Figure II. Circle Packing Model

Geometric Modeling of a Cylindrical Part using Circle Packing

- 2. Pack spheres in circular packing model, and sum the individual sphere interactions f_i relative to a planar surface at some distance (Figure III).



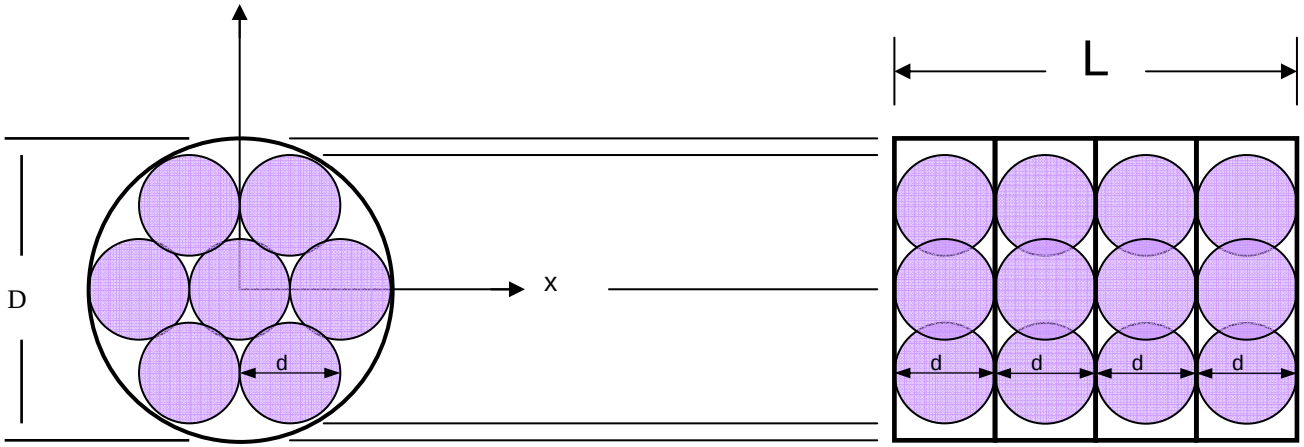
$$F_{vdW} \approx \sum_{i=1}^{n} f_i$$

Figure III. Summation of sphere to flat plane van der Waals force interactions..

Geometric Modeling of a Cylindrical Part using Circle Packing

- 3. Circular pattern represents a thin disk of depth *d,* total van der Waals interaction is approximately F_{vdW}.

- 4. Define a cylindrical part of length *L*, where *L* can be some multiple of *d.* *L = n*d.* Solve for *n*.

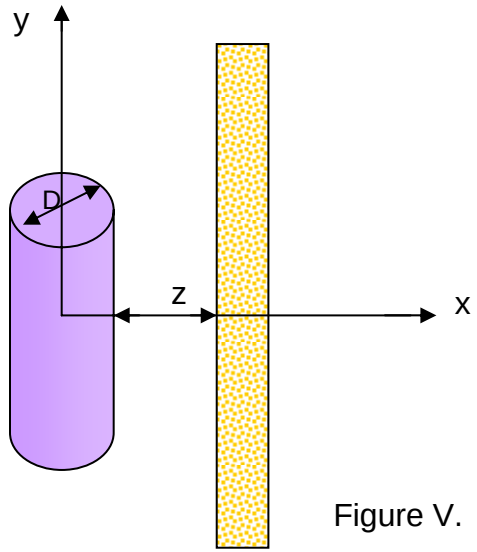


$F_{vdwcyl} = n * F_{vdW}$ $L = n * d$

Figure IV. Summation of sphere to flat plane van der Waals force interactions..

Geometric Modeling of a Cylindrical Part using Circle Packing

- 5. $F_{vdwcyl} = n*F_{vdw}$
- 6. Compare result to reference model (Figure V).



$$\frac{F_{vdw}}{lenght} = \frac{Ad^{1/2}}{16z^2}$$

Figure V. Van der Waals model of cylinder-flat plane.

- 7. Pack more spheres into cross section to obtain an optimal solution and then repeat steps 1 – 6.

Results

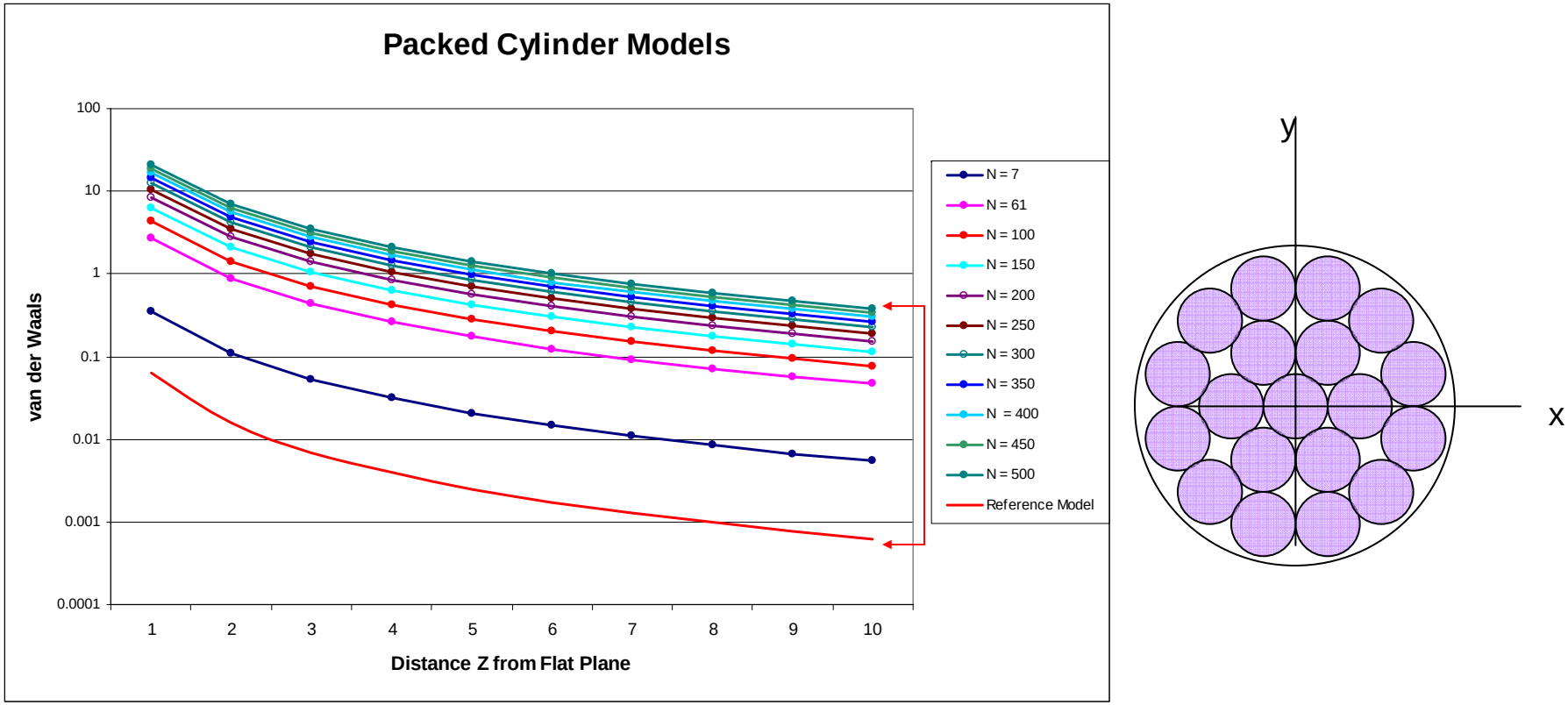


Figure VI. Van der Waals “sphere packed” finite model of cylinder-flat plane compared to reference model, where N represents the number of spheres packed into a given cross-section.

Conclusions & Future Direction:

1. Optimal sphere packing solution shows variation when compared to reference model. At best optimal solution only approximates reference model.

2. Future: model other van der Waals models such as sphere to sphere interactions using sphere packing . Desirable to establish a comparison to other known van der Waals solutions.

References

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- [2] Kitchener, J.A. “Surface Forces in the Deposition of Small Particles.” *Journal of the Society of Cosmetic Chemists.* Vol. 24 (1973): 709-725.
- [3] Bowling, R. Allen. “Theoretical Review of Particle Adhesion.” *Particles on Surfaces I*. K. L. Mittal, editor, (1988) 129-142.
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- [5] The best known packings of equal circles in the unit circle (up to *N* = 500) http://hydra.nat.uni-magdeburg.de/packing/cci/cci.html

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