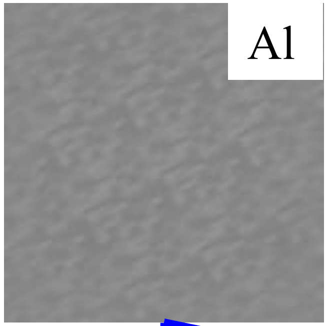
Cross-property connections in structural health monitoring

Cross-property connections



$E = 70\text{GPa}$

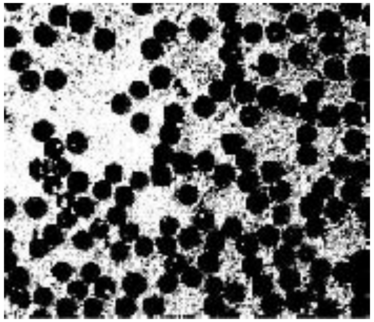
$\nu = 0.33$

$K_T = 146$ W/(mK)

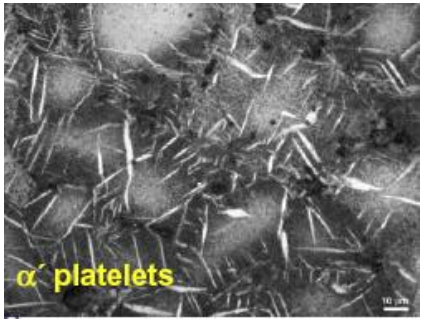
$R_e = 2.65\text{x}10^{-8}$-m

Change in all the material properties due to the microstructure

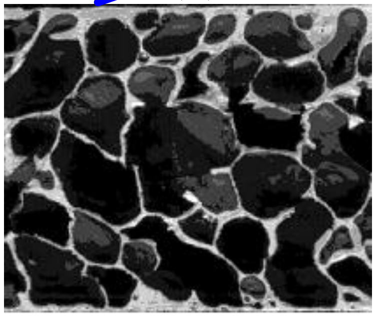
(1)



(2)



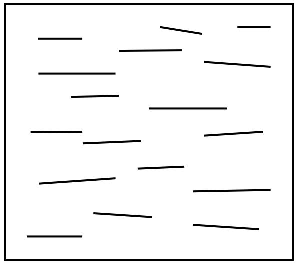
(3)



(1)Aluminum alloy reinforced with boron; (2) Radiation damage in aluminum alloy; (3)Aluminum foam

Cross-property connections in simplest cases

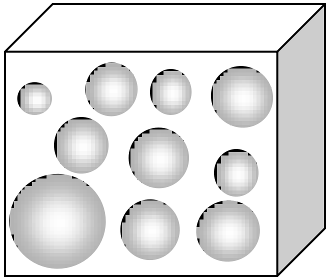
Microcracked anisotropic material



$$\frac{E_0 - E_i}{E_i} = \frac{4\left(1-\nu_0^2\right)}{2-\nu_0}\frac{k_0 - k_i}{k_i}$$

(Young's modulus *vs*. conductivity in any direction x_i)

Material with spherical voids

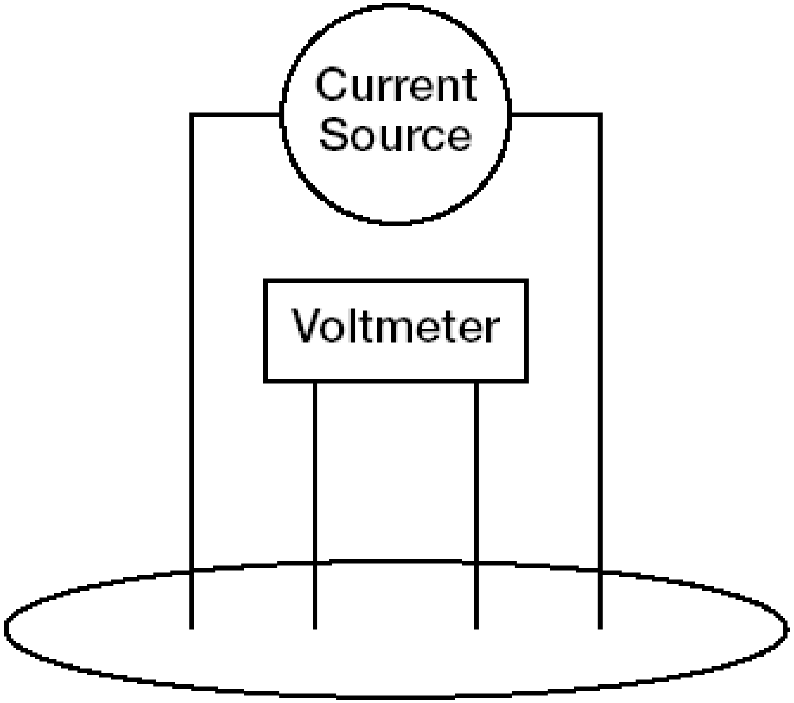


$$\frac{E_0 - E_{eff}}{E_{eff}} = \frac{\left(1-\nu_0\right)\left(9+5\nu_0\right)}{\left(7-5\nu_0\right)}\frac{k_0 - k_{eff}}{k_{eff}}$$

Why cross-property connections ?

Methodology is

- Non-destructive
- Simple
- Can be conducted *in-situ*
- Sufficiently accurate
- No cost consuming

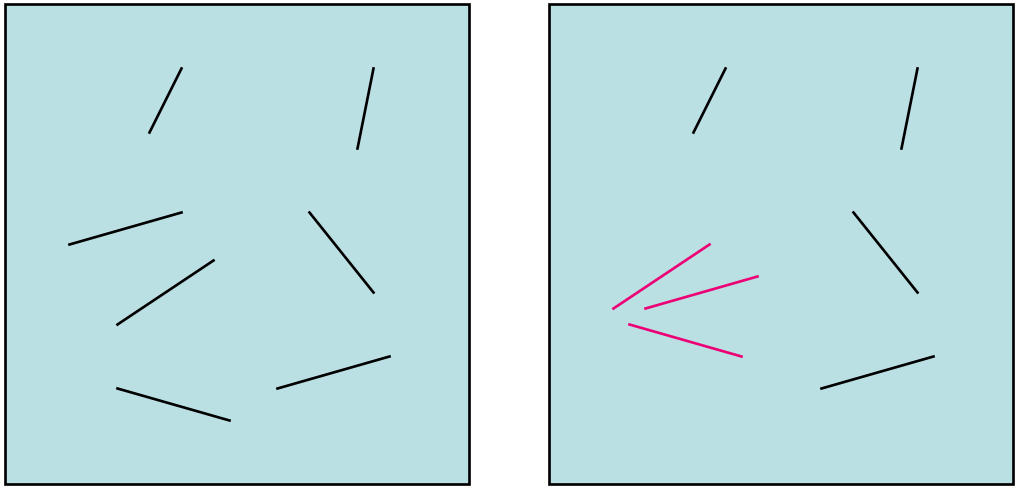


Four-point collinear probe resistivity configuration

Challenge

Both elastic and conductive properties are volume average.

Structural health monitoring problems are fracture related



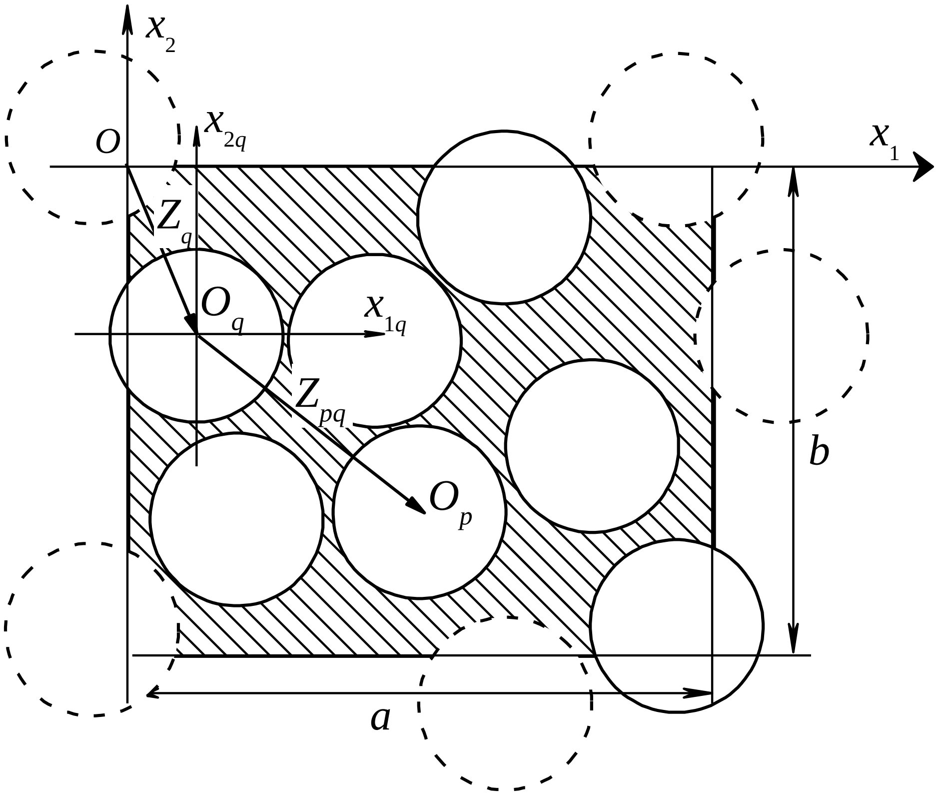
Sensitivity to clustering

is low, for the elastic and conductive properties

is high, for fracture

What are the proper microstructural parameters?
Is it possible to evaluate strength via electric conductivity?

Proper microstructural parameters

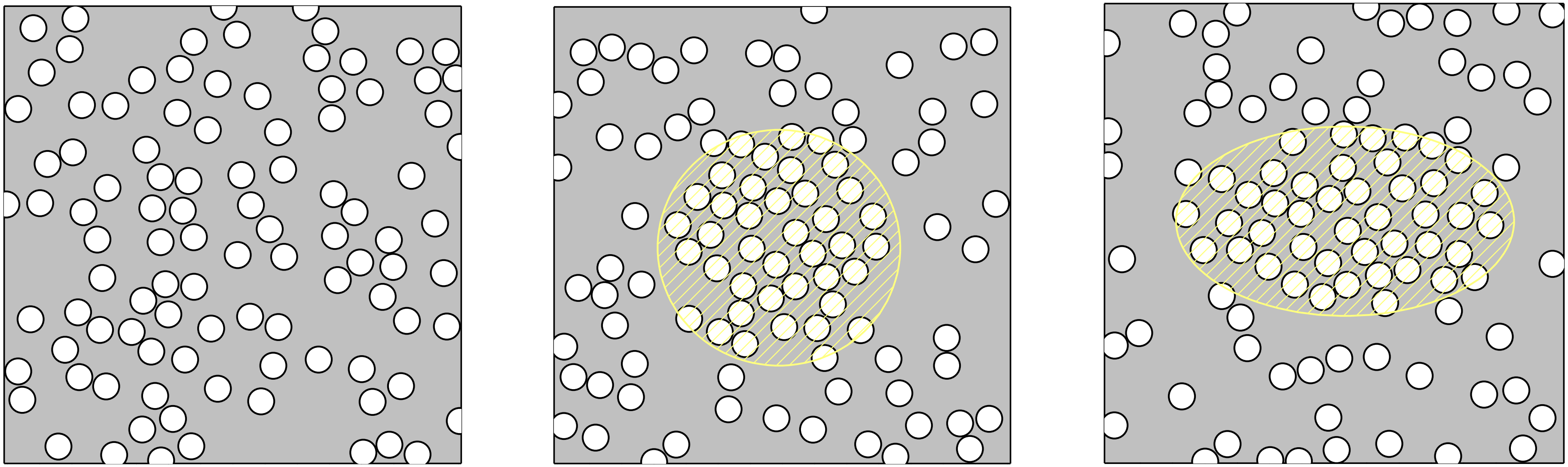


We study a quasi-random, or generalized periodic, microstructure with the periods a and b along the axes Ox_1 and Ox_2. The whole composite bulk can be obtained by translating the cell in two orthogonal directions.

Two types of defects have been studied:

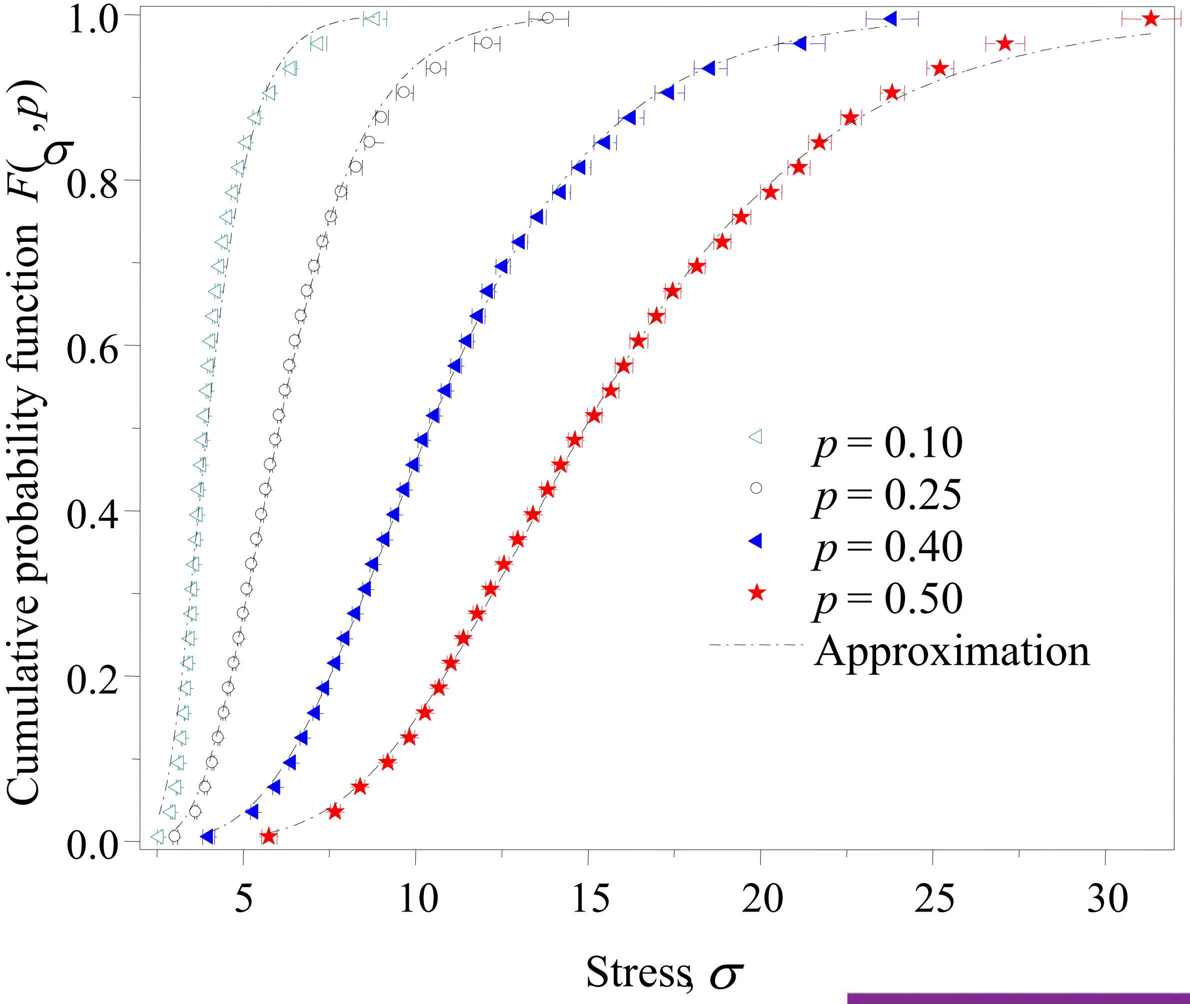
circular pores and cracks

Microstructures



To generate a quasi-random structure, the 2D version of the molecular dynamics (MD) algorithm of growing particles is utilized. Pore clusters were obtained from the uniform microstructure with the porosity equal to that inside the cluster. The pores outside the cluster are then removed randomly, one by one, until the prescribed overall porosity of the material is reached.

Statistics of the max local stress concentration



Materials with uniformly distributed pores

The resulting distributions of the peak stress values match perfectly by the Gumbel rule

$$F(x) = exp\{-exp[-k(x - x_c)]\}$$

Analytical expression for the probability function

Since $x_c \to 3$ $k \to \infty$ when $p \to 0$

$$x_c(p) = 1.69 + 1.27\, exp(p/0.227);$$
$$k(p) = -0.228 - 0.597\, ln\, p.$$

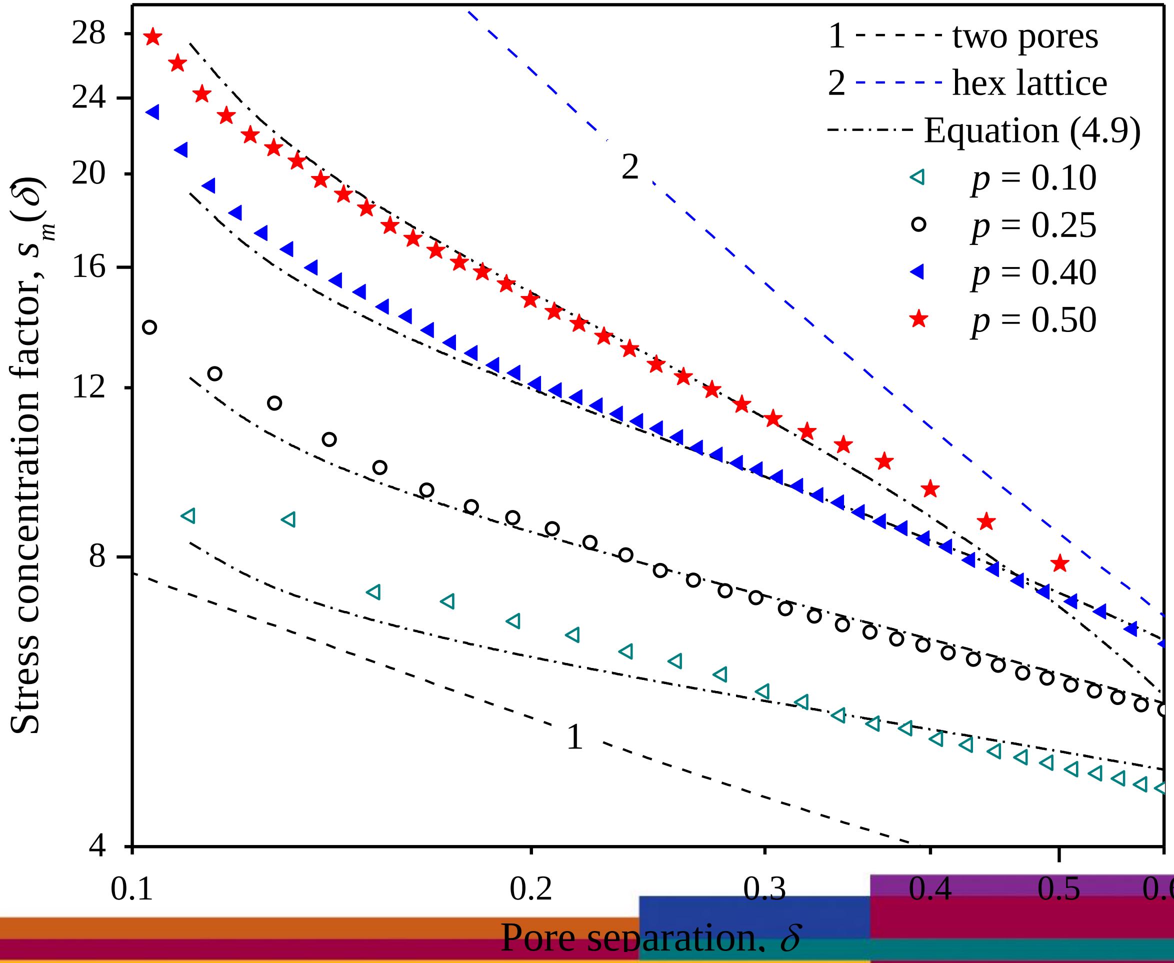
We use

$$F(\sigma, p) = \exp\left\{-\exp\left[-k(p)\left(\sigma - x_c(p)\right)\right]\right\}$$

as an analytical form of the cumulative probability function

$$F_{s_m}(\sigma) = P\left(s_m < \sigma\right)$$

Stress concentration factor as function of the distance between pores

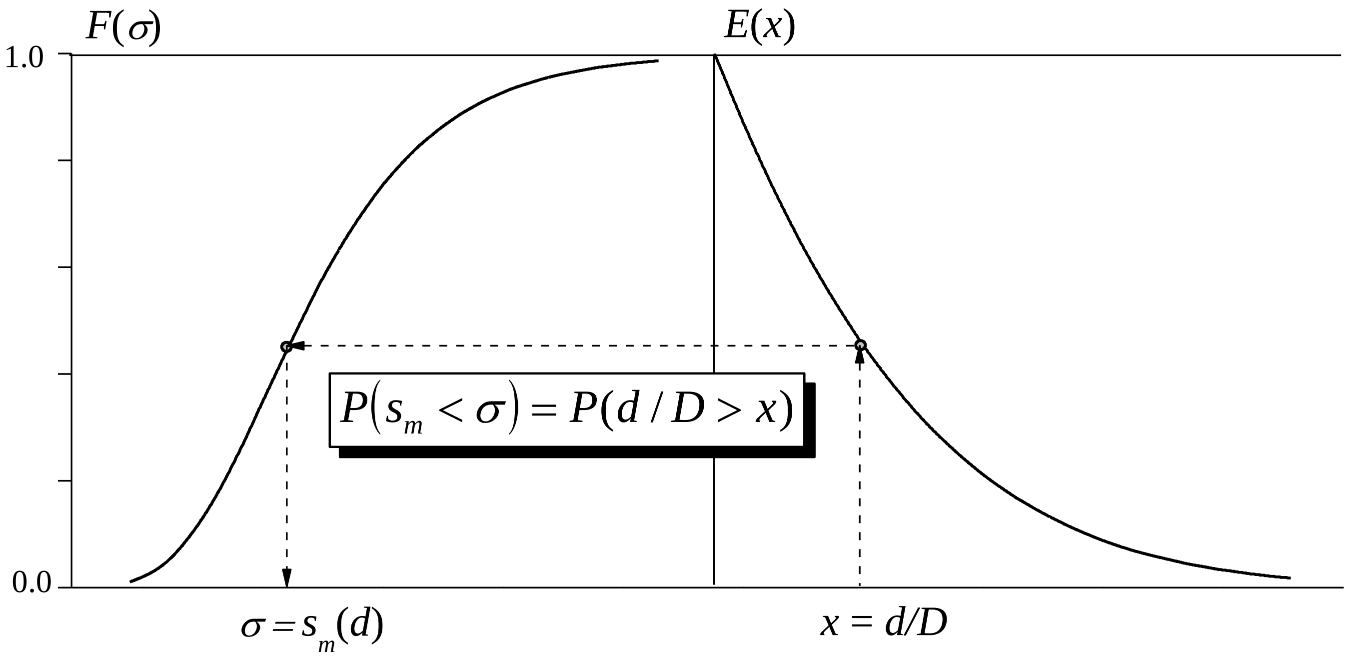


$$s_m(d,p) = x_c(p) - k^{-1}(p) \times \ln\left\{-\phi\left[a_0\left(k^2-1\right)+8a_1\left(k-1\right)\right]\right\},$$

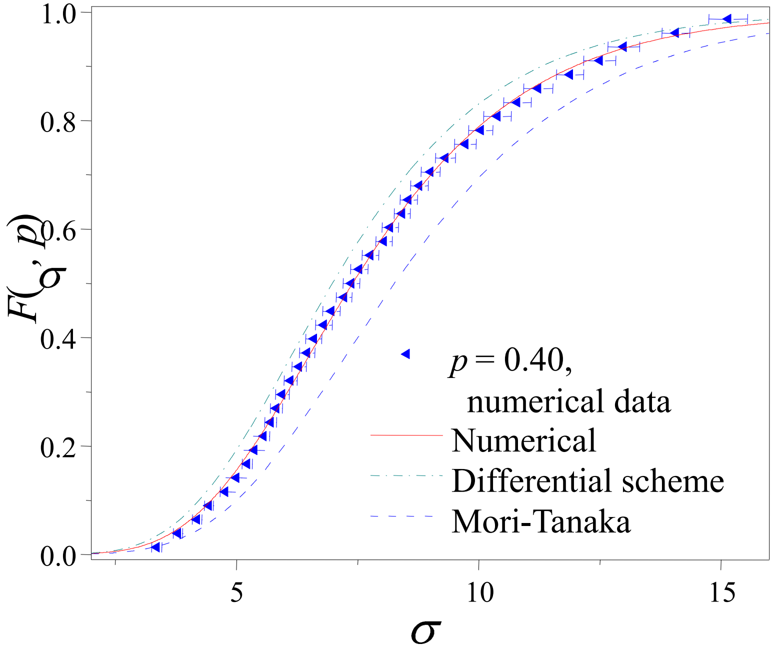
$$a_0 = (1+0.128p)/(1-p)^2$$

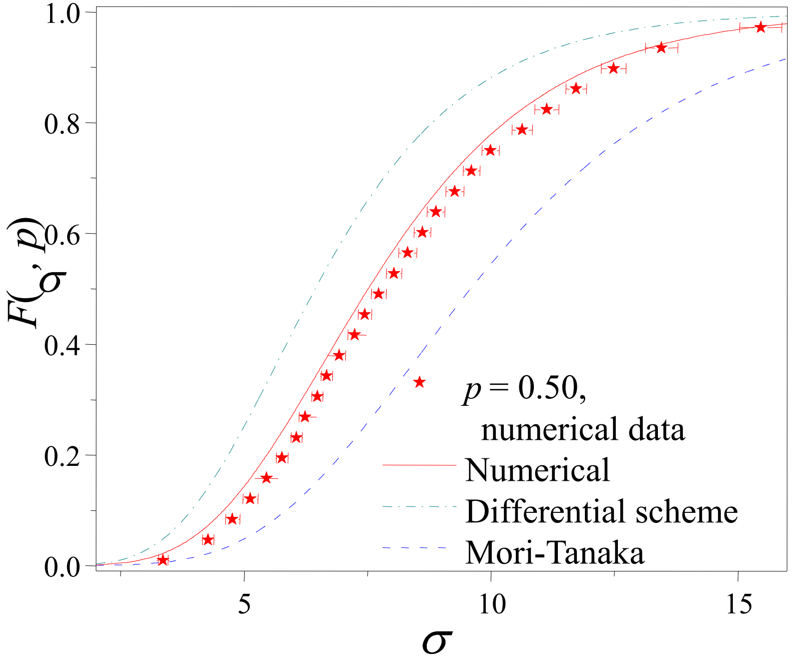
$$a_1 = -0.564p/(1-p)^2$$

Existence of this relationship means that probability of finding the pores at a distance larger than *d* is equal to the probability that maximal stress concentration in the material does not exceed a certain value σ



Pore cluster



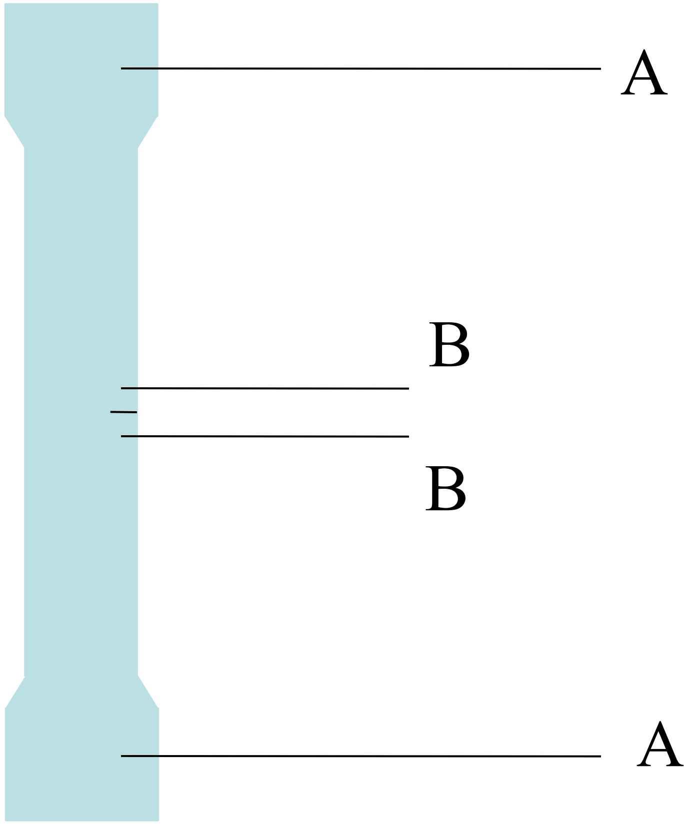


The probability depends on tensor of concentration and thus on the method of calculating the effective properties

An interesting observation is that simple one-particle schemes serve as bounds for real values of the peak stresses. More accurate values of the elastic constants (obtained numerically in our simulation) lead to a very good agreement with the results of numerical experiment.

- Distribution of maximal stresses in porous material follows Gumbel’s rule derived for statistics of extreme values
- Explicit correspondence between statistics of maximal stresses and statistics of nearest neighbors (Torquato, 1995) is observed.
- Analytical expression is obtained for statistics of maximal stresses in a porous material with arbitrary microstructure as a function of porosity and for statistics of minimal distances between the nearest neighbors.
- A simple model that describes statistics of local stresses inside clusters of circular or elliptic shape is proposed. Predictions according to this model are in good agreement with the numerical data.
- Spatial distribution of circular pores (uniform or with distinguishable pore clusters) produce negligible effect on the overall thermal and elastic properties (at least, in the examined interval of porosity variation).

Strength versus electrical conductivity (work in progress)



Two specimens have been compared: unloaded and after 50000 cycles of loading.

Difference in electrical conductivities:
AA - 10%   BB - 5 times!

Thus, for monitoring of the accumulated damage, we need to scan the “local conductivity”.

In other words we need to measure conductivity gradient.